Possible Framework for Using the Strut-and-Tie Method (STM) with Ultra-High Performance Concrete (UHPC)

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FOREWORD

Research related to ultra-high performance concrete (UHPC) has been ongoing for the past few decades. UHPC offers enhanced mechanical and durability properties that make it an ideal material for use in the construction, repair, and retrofit of our Nation's highway bridges. The use of UHPC for structural members will become more widespread with the release of the AASHTO *Guide Specifications for Structural Design with Ultra-High Performance Concrete*. However, there is limited information provided in the guide specifications and in the industry in general related to designing UHPC deep beam elements, which include many substructure members.

The information presented in this report provides background, context, and foundational knowledge to bridge owners, designers, and researchers interested in using UHPC for deep beam members. The report aims to give an initial proposed framework for using the strut-and-tie method for UHPC deep beam members for bridge owners, designers, and researchers.

Shay K. Burrows P.E.
Resource Center Director
Federal Highway Administration
Office of Innovation Implementation

Notice

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16. Abstract

This report provides initial thoughts on how the strut-and-tie method (STM) can be used with ultra-high performance concrete (UHPC) disturbed regions, such as those near supports and the points of application of concentrated loads at strength and extreme limit states. A brief background on STM and plastic design methodology is provided for context. Limitations for the use of STM for UHPC members are discussed. The report includes initial recommendations for using STM for UHPC members based on provisions in the AASHTO *Guide Specifications for Structural Design with UHPC* and highlights related research needed. An example problem (based on FHWA-NHI-17-071) is provided to show how the basic framework may be applied for UHPC deep beams.

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NOTATION

A_1	area of section between y_1 from the center of gravity and the outside face of the section; area immediately under bearing for confinement modification factor calculation (inch^2)
A_2	area surrounding the bearing node for calculating the confinement modification factor (inch^2)
A_c	area of concrete or UHPC (inch^2)
A_{cn}	area of node face (inch^2)
$A_{cn,CC}$	area of conventional concrete portion of node face (inch^2)
$A_{cn,UHPC}$	area of UHPC portion of node face (inch^2)
A_h	area of horizontal reinforcement within distance s_h (inch^2)
A_{ps}	total area of prestressing in tension tie (inch^2)
$A_{s,req}$	required area of steel (inch^2)
A_{st}	area of non-prestressed reinforcement in tension tie (inch^2)
A_{tie}	area of UHPC in tension tie (inch^2)
A_v	area of transverse reinforcement in vertical direction within distance s_v (inch^2)
a	shear span (inch)
a_f	distance from centerline of girder reaction to vertical reinforcement in backwall or stem of inverted T (inch)
b_w	width of member’s web (inch)
C	compression force (kips)
c	distance from centerline of bearing to end of beam ledge (inch)
c_c	clear cover (inch)
d	distance from extreme compression face to centroid of non-prestressed reinforcement (inch)
d_e	depth of center of gravity of steel; effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (inch)
d_f	distance from top of ledge to the bottom longitudinal reinforcement (inch)
d_p	distance from extreme compression face to centroid of prestressing strands (inch)
d_v	effective shear depth (inch)
E	modulus of elasticity (ksi)

E_c	modulus of elasticity of concrete or UHPC (ksi)
E_s	modulus of elasticity of steel reinforcement (ksi)
E_p	modulus of elasticity of prestressing strands (ksi)
f_2	stress in compressive strut (ksi)
f_c	stress in extreme compression (top) fiber; stress in concrete (ksi)
f_{cu}	limiting compressive stress at a node face (ksi)
$f_{cu,CC}$	limiting compressive stress for conventional concrete at a node face (ksi)
$f_{cu,UHPC}$	limiting compressive stress for UHPC at a node face (ksi)
$f_{c@0.0019}$	UHPC stress at strain of 0.0019 (ksi)
$f_{c@0.0023}$	UHPC stress at strain of 0.0023 (ksi)
f'_c	compressive strength of concrete for use in design (ksi)
f'_{ci}	compressive strength of concrete at release for use in design (ksi)
f_{pe}	effective stress in prestressing (ksi)
f_{pu}	ultimate strength of prestressing strands (ksi)
f_{py}	yield strength of prestressing strands (ksi)
f_s	stress in steel (ksi)
f_t	stress in extreme tension face, bottom (ksi)
$f_{t,cr}$	effective cracking strength for UHPC (ksi)
$f_{t,cri}$	effective cracking strength for UHPC at time of transfer (ksi)
$f_{t,loc}$	crack localization strength for UHPC (ksi)
f_y	yield strength of transverse reinforcement (ksi)
h	section height (inch)
h_a	height of tension tie (inch)
h_{STM}	distance between top horizontal of compression struts and bottom horizontal tension ties (inch)
L	span length (ft)
ℓ_a	available development length at section of interest (inch)
ℓ_b	bearing length (inch)
ℓ_d	development length for reinforcement (inch)

ℓ_{db} basic development length for reinforcement (inch)

ℓ_{dh} development length for hooked reinforcement (inch)

ℓ_{fiber} fiber length (inch)

ℓ_{hb} basic development length for hooked reinforcement (inch)

ℓ_t transfer length for reinforcement (inch)

M moment in member (kip-ft)

M_0 applied moment at ends of member (kip-ft)

$M_{m,tot}$ total moment at midspan due to w_1 and w_2 (kip-ft)

$M_{max,service}$ maximum moment from service loads (kip-ft)

$M_{max,strength}$ maximum moment from strength loads (kip-ft)

M_p plastic moment of section (kip-ft)

M_p^- plastic moment of section associated with negative bending (kip-ft)

M_p^+ plastic moment of section associated with positive bending (kip-ft)

M_u factored flexural demand (kip-ft)

m confinement modification factor

N axial force in member (kips)

N_u factored axial force demand in member (kips)

$n_{b,max}$ maximum number of bars across width of section

n_{req} required number of bars

P applied force (kips)

P_n nominal resistance of a node face or tie (kips)

P_s steel contribution to nominal resistance of tension member (kips)

$P_{service}$ applied service load (kips)

$P_{strength}$ applied factored load for strength limit state (kips)

P_{UHPC} UHPC contribution to nominal resistance of tension member (kips)

$R_{L,service}$ left reaction from service loads (kips)

$R_{L,strength}$ left reaction from strength loads (kips)

$R_{R,service}$ right reaction from service loads (kips)

$R_{R,strength}$ right reaction from strength loads (kips)

s spacing of transverse reinforcement (inch)

s_h spacing of horizontal crack control reinforcement (inch)

s_v spacing of vertical crack control reinforcement (inch)

T tension force (kips)

t_{bf} bottom flange thickness (inch)

t_{tf} top flange thickness (inch)

V_c nominal shear resistance of the concrete (kips)

V_{cr} estimated cracking resistance (kips)

$V_{max,service}$ maximum shear from service loads (kips)

$V_{max,strength}$ maximum shear from strength loads (kips)

V_p component of prestressing force in the direction of the shear force (kips)

V_s nominal shear resistance provided by transverse reinforcement (kips)

V_u factored shear demand (kips)

V_{UHPC} nominal shear resistance of the UHPC (kips)

$V_{UHPC,cr}$ estimated cracking resistance for UHPC member (kips)

v concrete efficiency factor

v_u shear stress due to factored shear force (kips)

W width of bearing plate or pad (inch)

W_E external virtual work (kip-ft)

W_I internal virtual work (kip-ft)

w distributed load (kip/ft)

w_1 distributed load causing first hinging (kip/ft)

w_2 distributed load causing second hinging (kip/ft)

$w_{(1)}$ distributed load at failure for assumed collapse mechanism (1) (kip/ft)

$w_{(2)}$ distributed load at failure for assumed collapse mechanism (2) (kip/ft)

w_c unit weight of concrete or UHPC (kcf)

w_s strut width (inch)

w_{tot}	total distributed load applied to member at the formation of a collapse mechanism (kip/ft)
x	distance from left support (ft)
y	distance from neutral axis (inch)
α	proportion of load going to the right support
α_u	reduction factor to account for the non-linearity of the UHPC compressive stress-strain response
γ_u	reduction factor to account for the variability in the UHPC tensile stress parameters
Δ	deflection (inch)
ε	general strain (inch/inch)
ε_c	strain in UHPC; net compressive strain in extreme compression fiber of the UHPC section (inch/inch)
ε_{cp}	elastic compressive strain limit (inch/inch)
ε_{cu}	ultimate compressive strain of conventional concrete or UHPC for use in design (inch/inch)
ε_{pu}	ultimate strain of prestressed reinforcement, typically referred to as total elongation (inch/inch)
ε_{py}	strain in prestressed reinforcement at yield (inch/inch)
ε_s	net tensile strain in the extreme tension steel (inch/inch)
ε_{su}	ultimate strain of non-prestressed reinforcement, typically referred to as total elongation (inch/inch)
ε_t	net tensile strain in extreme tension fiber of the UHPC section (inch/inch)
$\varepsilon_{t,cr}$	elastic tensile strain limit of UHPC corresponding to a tensile stress of $\gamma f_{t,cr}$ (inch/inch)
$\varepsilon_{t,loc}$	crack localization strain of UHPC for use in design (inch/inch)
ε_y	yield strain for the reinforcement (inch/inch)
θ	rotation in elastic beam theory (radians)
$\theta_{(1)}$	rotation in elastic beam theory for assumed collapse mechanism (1) (radians)
$\theta_{(2)}$	rotation in elastic beam theory for assumed collapse mechanism (2) (radians)
θ_s	strut angle (degrees)
λ	concrete density modification factor for conventional concrete

λ_{cf} coating factor

λ_{er} excess reinforcement factor

λ_{rc} reinforcement confinement factor

λ_{rl} reinforcement location factorϕ resistance factor

τ shear stress (ksi)

ψ curvature (rad/inch)

ψ_{ult} curvature associated with failure of section (rad/inch)

ψ_{y} curvature associated with yielding of reinforcement (rad/inch)

CHAPTER 1. INTRODUCTION

INTRODUCTION

Specifications for the structural design of ultra-high performance concrete (UHPC) members are provided in the AASHTO *Guide Specifications for Structural Design with UHPC*, hereafter referred to as UHPC Structural Design Guide. The UHPC Structural Design Guide provides specifications and guidance related to UHPC material properties, limit state design methodologies, design of B-regions for flexure, shear, and torsion, and prestressed and conventional reinforcement. The UHPC Structural Design Guide offers limited guidance on the design of D-regions.

The design of D-regions with UHPC is introduced in UHPC Structural Design Guide Article 8.

> *Refined analysis, strut-and-tie, and elastic stress analysis methods may be used to determine the internal force effects in disturbed regions, such as those near supports, and the points of application of concentrated loads at strength and extreme limit states. The design method shall be approved by the owner and shown in the contract documents.*

There are two specific limitations delineated in the article:

- Internal strains in the UHPC and reinforcement at strength and extreme limit states shall use material properties in UHPC Structural Design Guide Article 4. Tensile strain in the UHPC shall be limited to $\gamma_u \varepsilon_{t,loc}$.
- Calculated resistances for complex D-regions should be justified by performance testing a prototype.

No additional guidance is provided in the UHPC Structural Design Guide at this time.

OBJECTIVES OF REPORT

The design of UHPC substructure elements and some components for UHPC superstructure elements may require the use of the strut-and-tie method (STM). The UHPC Structural Design Guide allows for the use of STM for the design of these elements with approval of the owner. The purpose of this document is to provide owners and designers with context, guidance, and constraints for the use of STM with UHPC until further research and recommendations can be developed.

AASHTO LRFD BDS Article 5.8 is not necessarily applicable to UHPC and should not be directly used for the design of UHPC components. Some of the limitations of the current state of knowledge related to using STM for UHPC members are discussed in this report.

There is guidance on UHPC behavior and limitations in the UHPC Structural Design Guide that may be applicable to STM when applied to UHPC members. This guidance is highlighted in this document to give some initial thoughts on how STM may be engaged when designing UHPC members.

The preliminary recommendations contained in this report are intended to be conservative, recognizing that further research and use may identify opportunities for revision and optimization of the proposed design approaches. Researchers, designers, and owners are encouraged to further investigate these recommendations.

LIMITATIONS OF REPORT

There are several limitations to the contents of this report and the use of STM for UHPC components in general, including the following:

- The design of UHPC components is strain dependent with the crack localization strain typically limiting the capacity. STM is not traditionally a strain-based approach. The use of the tension member provisions in UHPC Structural Design Guide Article 6.6 likely accounts for strain limitations for STM tension ties. However, consideration should also be given to other possible implications of strain limitations.
- The behavior of UHPC tension elements is still being investigated. Until more research data becomes available, a UHPC tension tie design should follow UHPC Structural Design Guide Article 6.6.1 in which reduction factors on the UHPC tensile properties are introduced, a minimum amount of tension reinforcement ($P_s \geq 0.8P_n$) is prescribed, and unreinforced tension ties are prohibited.
- Selection of an appropriate model that closely aligns with the actual distribution of stresses is important for creating an efficient design using STM for conventional concrete members. A model that greatly diverges from the actual stress distribution in conventional concrete members will still lead to a safe design but may lead to excessive cracking in the member. This may also be true for UHPC members, but more research is needed to explore this behavior. STM for UHPC should initially be limited to members where the model is known to closely align with actual stress distributions.
- STM is dependent on the ability of stresses to redistribute as cracking occurs. UHPC elements designed using STM should have the ability to redistribute stresses, i.e., they should be solid elements. STM should not be used to design UHPC trusses or lattice structures.
- The behavior of UHPC under biaxial and triaxial tension is still unknown. Biaxial and triaxial tension will likely decrease the crack localization strain and strength. The effect of this behavior on intersecting tension ties is unknown. The design of UHPC members with intersecting tension ties should be avoided until more is known about this behavior.
- The behavior of UHPC subjected to different load cases that create intersecting oblique or perpendicular cracks is still unknown. The recommendations in this report should not be used to design components anticipated to experience these load cases.
- UHPC Structural Design Guide Article 1.4 states that the provisions "were not developed to address the special considerations and detailing inherent in post-tensioned structures." The recommendations in this report should not be used for PT anchorage zones at this time until further research is conducted.

This listing of limitations should not be considered all-inclusive. Instead, owners and designers should recognize that a design process such as STM when combined with a new structural material like UHPC requires careful consideration of all assumptions in the model and performance limits of the material.

RESEARCH NEEDS

Research related to the use of STM for UHPC members is needed. The recommendations provided in this report are a starting point for considering the use of STM for UHPC members and also highlight the need for research in specific areas.

REPORT OVERVIEW

This report provides a brief background on the strut-and-tie method (STM) and its use with UHPC members. Brief backgrounds on plasticity theorems and plastic design in general are provided to help define what it means that STM is a lower-bound plasticity theorem and the inherent level of conservatism built into the design approach. An example based on Design Example 1 in FHWA-NHI-17-071 is provided in the Appendix to show how the framework presented in this report may be applied to a UHPC deep beam.

CHAPTER 2. BACKGROUND ON UHPC AND STM

INTRODUCTION

This chapter provides a summary of several different topics to give background and context for the recommendations provided in Chapter 3. A summary of the material properties for UHPC is first provided as the unique material properties of UHPC will affect the way STM should be used with UHPC elements. STM should be used for design of D-regions in concrete members; B-regions and D-regions are defined with examples of elements requiring STM for design. STM is a lower-bound plasticity theorem design approach; details are provided to explain what this means for designing using STM. STM modeling and definitions and basic design criteria for struts, ties, and nodes are also summarized.

BASIC UHPC MATERIAL PROPERTIES

UHPC is a cement-based composite material with high compressive and tensile strength, ductility and durability. The basic material properties for UHPC to use in structural design are specified in UHPC Structural Design Guide. The minimum material properties for UHPC are defined in UHPC Structural Design Guide Article 1.1 as the following.

- Compressive strength: $f'_c \geq 17.5$ ksi
- Effective cracking strength: $f_{t,cr} \geq 0.75$ ksi
- Crack localization strength: $f_{t,loc} \geq f_{t,cr}$
- Crack localization strain: $\varepsilon_{t,loc} \geq 0.0025$

Additional limits on design UHPC strengths at time of prestressing for pretensioned members and time of initial loading for non-prestressed members are specified in UHPC Structural Design Guide 9.1.2.

- Compressive strength at time of prestressing/loading: $f'_{ci} \geq 14.0$ ksi
- Effective cracking strength at time of prestressing/loading: $f_{t,cri} \leq 0.75 f_{t,cr}$ when $f'_{ci} \leq 0.90 f'_c$ (unless determined by physical tests, as approved by owner)

The compressive strength of UHPC is measured using ASTM C1856. The tensile response of UHPC is measured using the AASHTO *Standard Method of Test for Uniaxial Tensile Response of Ultra-High Performance Concrete* (T 397-22).

The design of UHPC members requires using stress-strain relationships for the materials in compression and tension. UHPC is assumed to have an elastic-plastic stress-strain response in compression, shown in Figure 1 (a).

UHPC compression stress-strain response: $f_c = \varepsilon_c E_c$ for $\varepsilon_c \leq \varepsilon_{cp}$

$f_c = \alpha_u f'_c$ for $\varepsilon_{cp} < \varepsilon_c \leq \varepsilon_{cu}$

The strain at the end of the elastic region in compression can be calculated using the modulus of elasticity for the UHPC (E_c).

Elastic compressive strain limit: $\varepsilon_{cp} = \alpha_u f'_c / E_c$

UHPC Structural Design Guide Eqn. 4.2.4.2-1

The ultimate compressive strain used in design, ε_{cu}, is the greater of ε_{cp} and 0.0035, unless determined by physical tests. The ultimate compressive strain is the strain corresponding to the compressive strength, f'_c, recorded during a compression test performed in accordance with ASTM C1856 (see UHPC Structural Design Guide Article 4.2.4.2).

The reduction factor for compressive strength, α_u, accounts for the nonlinearity of the stress-strain response and shall not be greater than 0.85. Unless additional compressive strength testing shows otherwise, the reduction factor $\alpha_u = 0.85$ may be used.

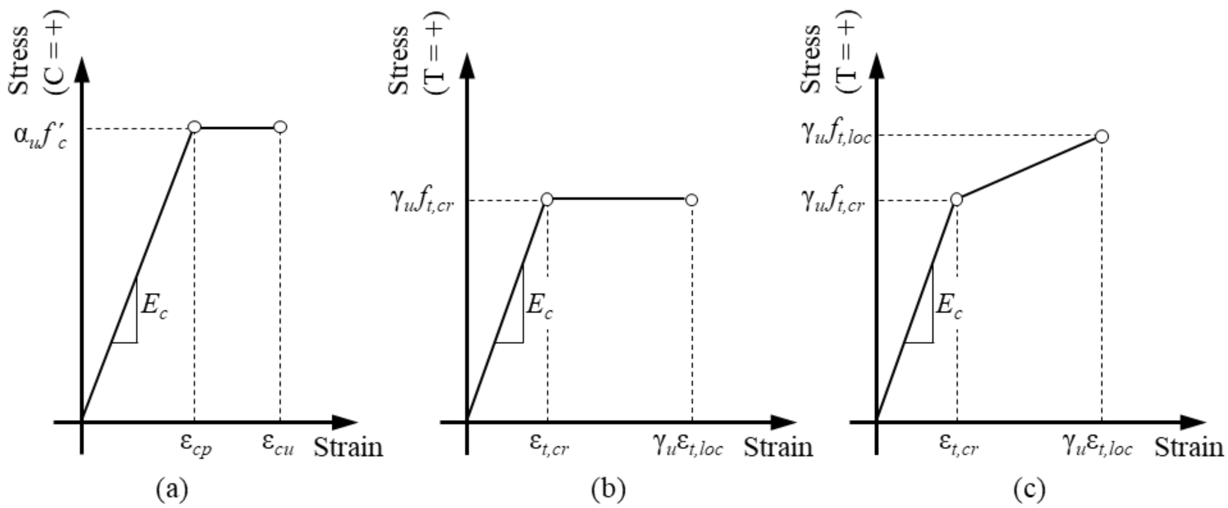

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Figure 1. Illustration. Idealized UHPC stress-strain curves for (a) compression, (b) tension if $f_{t,loc} < 1.20 f_{t,cr}$ and (c) tension if $f_{t,loc} \geq 1.20 f_{t,cr}$.

The tensile stress-strain response for UHPC is defined in UHPC Structural Design Guide Article 4.2.5.4 and is idealized either as elastic-plastic (if $f_{t,loc} < 1.20 f_{t,cr}$) or bilinear (if $f_{t,loc} \geq 1.20 f_{t,cr}$), as shown in Figure 1 (b) and (c).

UHPC tension stress-strain response (if $f_{t,loc} < 1.20 f_{t,cr}$):

$$f_c = \varepsilon_c E_c \text{ for } \varepsilon_c \leq \varepsilon_{t,cr}$$

$$f_c = \gamma_u f_{t,cr} \text{ for } \varepsilon_{t,cr} < \varepsilon_c \leq \gamma_u \varepsilon_{t,loc}$$

UHPC tension stress-strain response (if $f_{t,loc} \geq 1.20 f_{t,cr}$):

$$f_c = \varepsilon_c E_c \text{ for } \varepsilon_c \leq \varepsilon_{t,cr}$$

$$f_c = \gamma_u f_{t,cr} + (\varepsilon_c - \varepsilon_{t,cr})(\gamma_u f_{t,loc} - \gamma_u f_{t,cr})/(\gamma_u \varepsilon_{t,loc} - \varepsilon_{t,cr}) \text{ for } \varepsilon_{t,cr} < \varepsilon_c \leq \gamma_u \varepsilon_{t,loc}$$

The cracking strain is calculated using the effective cracking strength and stiffness of the UHPC.

Effective cracking strain: $\varepsilon_{t,cr} = \gamma_u f_{t,cr} / E_c$

The reduction factor for tension strength, γ_u, shall not be greater than 1.0. This factor may be used to account for potential undesirable fiber orientation (UHPC Structural Design Guide Article 4.2.5.4). Unless additional tension testing shows otherwise, the reduction factor $\gamma_u = 1.0$ may be used.

Additional details on the mechanical properties for UHPC can be found in several other resources (Helou et al. 2022, Haber et al. 2018).

BERNOULLI- AND DISTURBED-REGIONS

Typical sectional behavior is based on the Bernoulli hypothesis that axial strains vary linearly across the depth of a member, i.e., plane sections remain plane, as illustrated in Figure 2. The slope of the strain profile is assumed equal to the curvature, ψ, with top and bottom fiber strains of ε_c and ε_t, respectively, and zero strain at the neutral axis (n.a.).

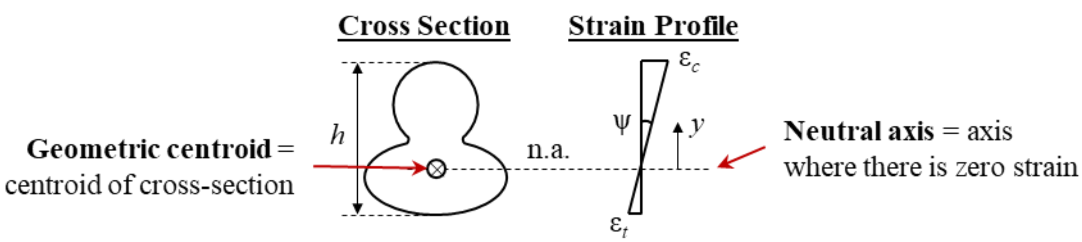

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Figure 2. Illustration. Assumed plane sections remain plane across the depth.

Regions that exhibit this behavior are called B-regions (Bernoulli or beam regions) and can be reasonably designed using sectional design approaches. If a beam is subjected to pure moment, strains are assumed to vary linearly across the depth of the member along the entire length of the member, as shown in Figure 3. The entire beam in this case would be considered a B-region.

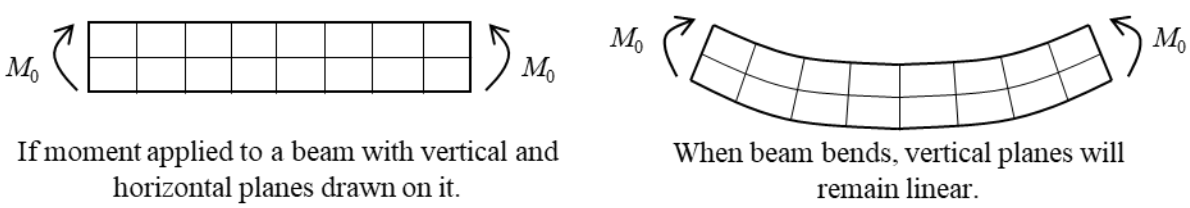

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Figure 3. Illustration. Typical strain profile along the length of a beam with pure moment applied.

A region is considered a D-region (disturbed or deep beam region) when the strain does not vary linearly across the depth of the section. D-regions are generally assumed to extend a distance equal to the tension steel depth, *d*, away from any load point, support, or geometric discontinuity in a member. An example of a typical distinction between B- and D-regions is shown in Figure 4 with a typical stress flow for a simply supported beam with point load toward one support.

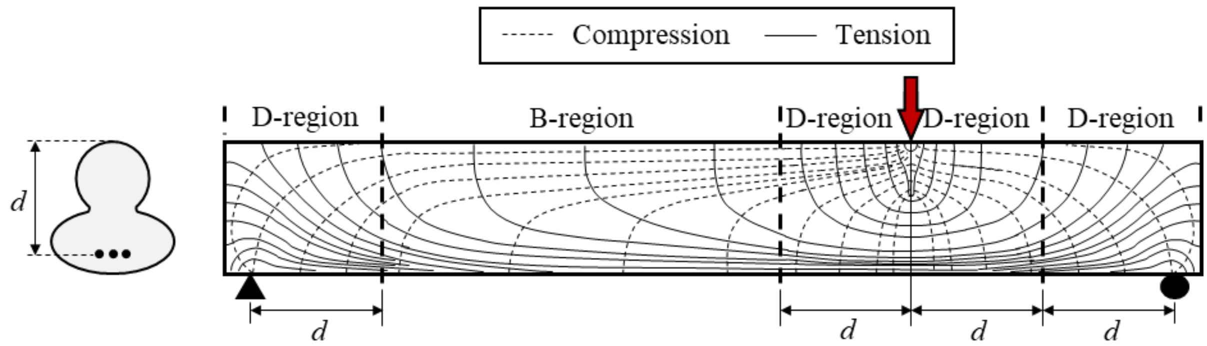

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Figure 4. Illustration. Typical stress flow in simply supported beam with point load toward one support with B- and D-Regions highlighted, based on figure from Schlaich et al., 1987.

Sectional design approaches are not valid in regions where strains do not vary linearly across the section depth (D-regions); the use of sectional design approaches in these regions can lead to unconservative designs. STM or empirically calibrated design approaches (e.g., specifications outlined in AASHTO LRFD BDS Article 5.8.4) should be used in D-regions. Typical members with distributed loads and HL-93 loading are generally designed assuming the entire member is a B-region using conventional sectional design approaches. STM or empirically calibrated design approaches are generally reserved for specific types of members or components of members. A sample of D-regions in structural elements typically designed using STM is shown in Figure 5.

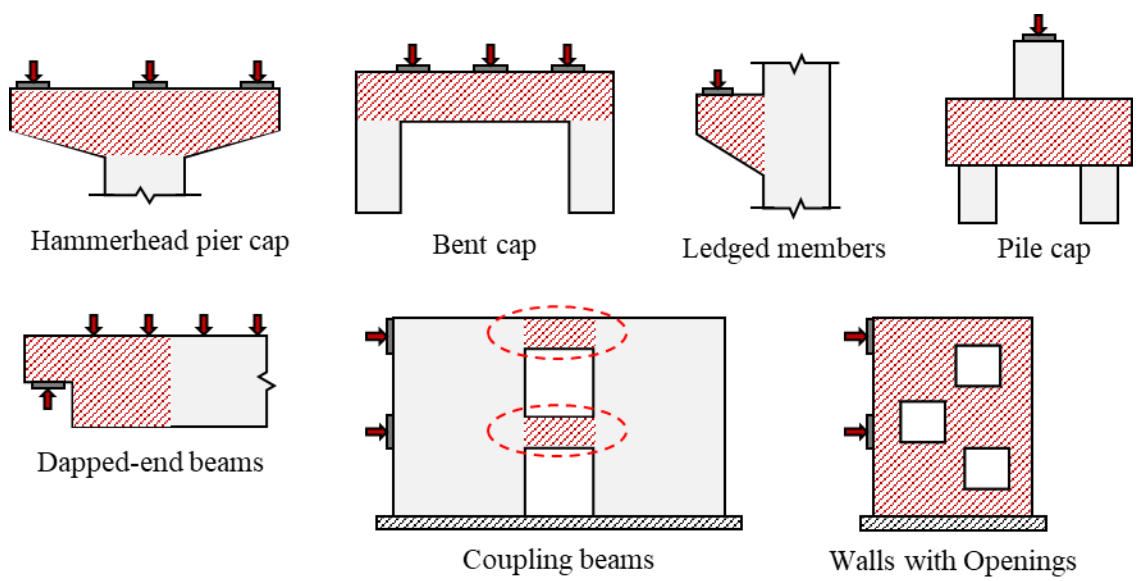

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Figure 5. Illustration. Example of D-regions (dashed regions) in structural elements.

PLASTICITY THEOREMS AND LIMIT STATE DESIGN

STM is a lower-bound plasticity theorem-based design approach. A basic understanding of plasticity theorems is important for understanding the inherent conservatism built into STM.

Explanations for plasticity theorems can be complex, but several basic points are outlined in this section.

Plastic design approaches consider the ultimate limit state of a structure. This means:

- While there may be accompanying design equations that address serviceability concerns, plastic design approaches themselves do not consider the behavior of the structure under the service limit state.
- Plastic design approaches generally consider inelastic material properties (stress and strains) and post-elastic behavior of a member (e.g., hinging, redistribution of stresses, redistribution of moments). The inelastic behavior for materials will affect the plastic capacity of a member.
- Plastic design approaches consider the capacity of a member or structure, not just comparing capacity to demand at individual sections along the length. Redistribution of stresses and moments is considered in plastic design approaches. The estimated capacity corresponding to the collapse of the member or structure, not just failure at a single section, is calculated.

Plastic design approaches typically are evaluated based on three criteria:

- **Equilibrium**: A member is statically admissible if internal forces are in equilibrium with the applied loads and reactions.
- **Safe redistribution of stresses**: The stress field does not exceed the failure condition at any location. This is often called the yield criterion although it is not necessarily associated with the yielding of reinforcement.
- **Collapse mechanism**: The distribution of stresses results in a collapse mechanism for the structure. The structure is at a point where it can undergo additional deflection without resulting in an increased load (i.e., kinematic instability).

General plastic design approaches are often illustrated using indeterminate beams where the redistribution of moments is required to form a collapse mechanism, like that shown in Figure 6 (a). The sectional behavior of the member is typically assumed to be elastic-plastic for simplicity, like that shown in Figure 6 (b). For this example, the positive and negative moment capacity will be assumed to be equal: $M_p = M_p^+ = M_p^-$.

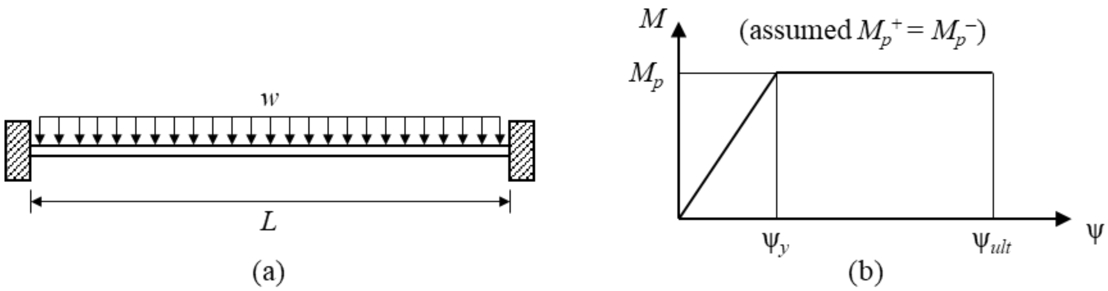

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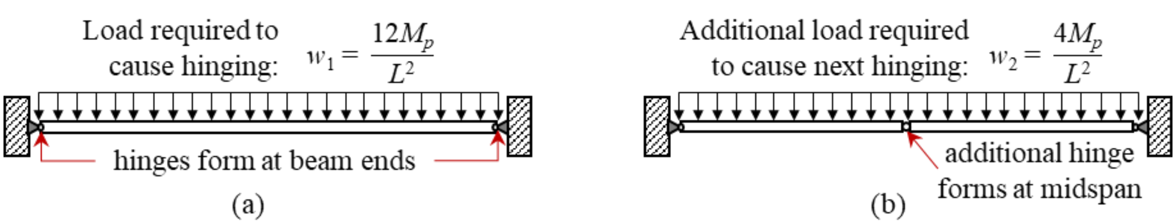
Figure 6. Illustration. Typical example used to illustrate plastic design approaches, (a) fixed-fixed beam with distributed load and (b) assumed elastic-plastic moment-curvature response.

In this case, equilibrium is always satisfied as long as the analysis for M_p is done correctly. The plastic design is considered safe (i.e., yield criterion satisfied) as long as the applied moment along the length is less than the plastic moment. A collapse mechanism occurs if enough plastic hinges are formed to result in a kinematically unstable structure.

Uniqueness Theorem (Estimated Capacity = Actual Capacity)

For the fixed-fixed beam example, a distributed load resulting in the beam end moments equal to the plastic moment capacity, M_p, of the beam will result in hinges forming at the ends of the beam, shown in Figure 7 (a). The end moments for a fixed-fixed beam are equal to $wL^2/12$. Setting this moment equal to the plastic moment capacity of the beam and solving for the distributed load will give the load required to cause first hinging: $w_1 = 12M_p / L^2$. The midspan moment at this point is equal to $w_1L^2/24$ which is equivalent to $0.5M_p$.



Source: FHWA.

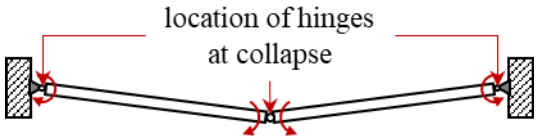
Figure 7. Illustration. Load required to cause (a) first hinging and (b) additional hinging resulting in collapse mechanism for a fixed-fixed beam with distributed load.

The beam will then behave as a simply supported beam until the formation of an additional hinge at midspan, shown in Figure 7 (b). This hinge will occur when the total moment at midspan is equal to the plastic moment capacity of the beam.

Total moment at midspan: $M_{m,tot} = (w_2L^2 / 8) + 0.5M_p = M_p$

Solving for the additional distributed load to cause next hinging results in $w_2 = 4M_p / L^2$.

The formation of the hinge at midspan results in a kinematically unstable structure, which is the definition of a collapse mechanism, as shown in Figure 8. The total load required to cause the collapse mechanism is the summation of w_1 and w_2, $w_{tot} = 16M_p / L^2$.



Source: FHWA.

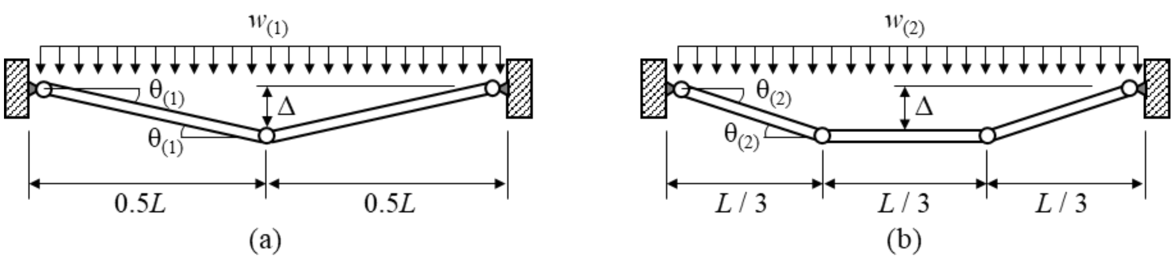
Figure 8. Illustration. Formation of collapse mechanism in fixed-fixed beam.

At this point in the example, all three criteria are satisfied: equilibrium, $M < M_p$ along the entire length, and a collapse mechanism is formed. This case would be considered to satisfy the uniqueness theorem, i.e., the load calculated to cause collapse is equal to the actual capacity of the member.

Upper-Bound Plasticity Theorem (Estimated Capacity ≥ Actual Capacity)

An alternate approach for calculating the plastic capacity of a structure is using the kinematic approach. For this method, the capacity is calculated using the principle of virtual work on an assumed failure mechanism.

Two different assumed collapse mechanisms for a fixed-fixed beam with distributed load are shown in Figure 9. The first assumed collapse mechanism assumes hinging at the beam ends and at midspan, shown in Figure 9 (a). The second assumed collapse mechanism assumes hinging at the beam ends and at the third points of the beam, shown in Figure 9 (b).



Source: FHWA.

Figure 9. Illustration. Two different assumed collapse mechanisms for a fixed-fixed beam with distributed load.

The applied load that is associated with the collapse mechanism is calculated by setting the internal virtual work equal to the external virtual work. These are as follows for the first assumed collapse mechanism shown in Figure 9 (a). Small angles are assumed for this example.

Rotation of hinges for first assumed collapse mechanism: $\theta_{(1)} = \Delta / (0.5L) = 2\Delta / L$

Internal work for first assumed collapse mechanism: $W_I = 4\theta_{(1)}M_p = 8\Delta M_p / L$

External work for first assumed collapse mechanism: $W_E = w_{(1)}\Delta(0.5L)$

Equilibrium condition for first assumed collapse mechanism: $W_I = W_E \rightarrow w_{(1)} = 16M_p / L^2$

This is equal to the capacity found above for the uniqueness theorem, which is a result of the assumed collapse mechanism being the same as the actual collapse mechanism.

The rotation, internal work, external work, and estimated capacity associated with the second assumed collapse mechanism shown in Figure 9 (b) are as follows.

Rotation of hinges for second assumed collapse mechanism: $\theta_{(2)} = 3\Delta / L$

Internal work for second assumed collapse mechanism: $W_I = 4\theta_{(2)}M_p = 12\Delta M_p / L$

External work for second assumed collapse mechanism: $W_E = w_{(2)}\Delta(2L / 3)$

Equilibrium condition for second assumed collapse mechanism: $W_I = W_E \rightarrow w_{(2)} = 18M_p / L^2$

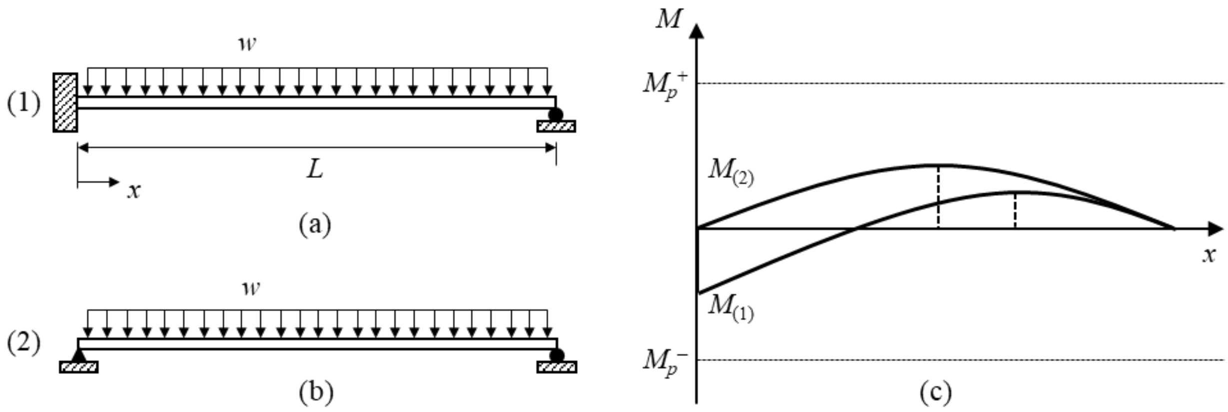
The capacity for the second assumed failure mechanism, $w_{(2)} = 18M_p / L^2$, is higher than the actual capacity, $w = 16M_p / L^2$, because hinging was assumed at the wrong locations. For this second assumed failure mechanism, equilibrium is satisfied, $W_I = W_E$, and a collapse mechanism is formed, but the yield criterion is not satisfied, since $M > M_p$ will occur along the length.

The kinematic method is an example of an upper-bound plasticity theorem approach since its use can result in an estimated capacity greater that the actual capacity of a structure. The yield line method sometimes used for slabs and concrete railing design (AASHTO LRFD Article A13.3.1) is an example of an upper-bound theorem approach.

Lower-Bound Plasticity Theorem (Estimated Capacity ≤ Actual Capacity)

A lower-bound plasticity theorem approach is one in which equilibrium is satisfied and the yield criterion is satisfied, but a collapse mechanism may not be formed under the estimated failure condition. This means that the estimated capacity found using a lower-bound plasticity theorem approach will always be less than or equal to the actual capacity of a member or system.

An example of a lower-bound plasticity theorem consideration for a fixed-pinned beam configuration is shown in Figure 10, based on an example provided by Muttoni et al. (1997). The fixed-pinned beam, shown in Figure 10 (a), has one degree of indeterminacy. The beam could conservatively be designed assuming that one of the restraints was released, e.g., the fixed-pinned beam could conservatively be designed as a pinned-pinned beam, as shown in Figure 10 (b). The moment diagrams for the three different cases are shown in Figure 10 (c). There would likely be excessive cracking if the beam were designed using one of the alternate configurations, but the capacity of the beam would be safe.



Source: FHWA.

Figure 10. Illustration. Example of lower-bound plasticity theorem consideration for fixed-pinned beam configuration.

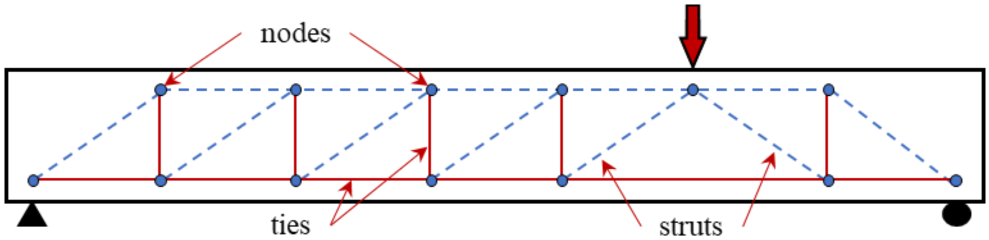
STM is a lower-bound plasticity theorem approach and will therefore always result in an estimated capacity equal to or less than the actual capacity of a member or system. Like the example, if the model does not align with actual stress distributions, there may be excessive cracking, but the design would be safe.

STRUT-AND-TIE METHOD (STM)

STM is a lower-bound plasticity approach in which the flow of stresses through a structure is modeled using a collection of compression elements (struts) and tension elements (ties) intersecting at nodes.

Strut-and-Tie Model

A strut-and-tie model is a theoretical truss model that represents the flow of stresses through the structure. Compressive stresses are consolidated into compression truss elements (struts) with tension elements (ties) resolving force equilibrium at the joints (nodes). A sample strut-and-tie model is shown in Figure 11; the theoretical flow of stresses for this beam are shown in Figure 4. STM should be used for D-regions, but it may also be used to design B-regions.



Source: FHWA.

Figure 11. Illustration. Sample strut-and-tie model for beam shown in Figure 4.

Development of a strut-and-tie model that accurately reflects the actual flow of stresses through a member or structure is important. Details on developing efficient strut-and-tie models are provided in AASHTO LRFD Article 5.8.2 with example models available for most deep beam elements in different reports and guide documents. Some of these include the following.

- Colorito et al. 2017. *Strut-and-Tie Modeling (STM) for Concrete Structures, Design Examples*, Report No. FHWA-NHI-17-071.
- ACI SP-273: *Further Examples for the Design of Structural Concrete with Strut-and-Tie Models*.
- ACI SP-208: *Examples for the Design of Structural Concrete with Strut-and-Tie Models*.

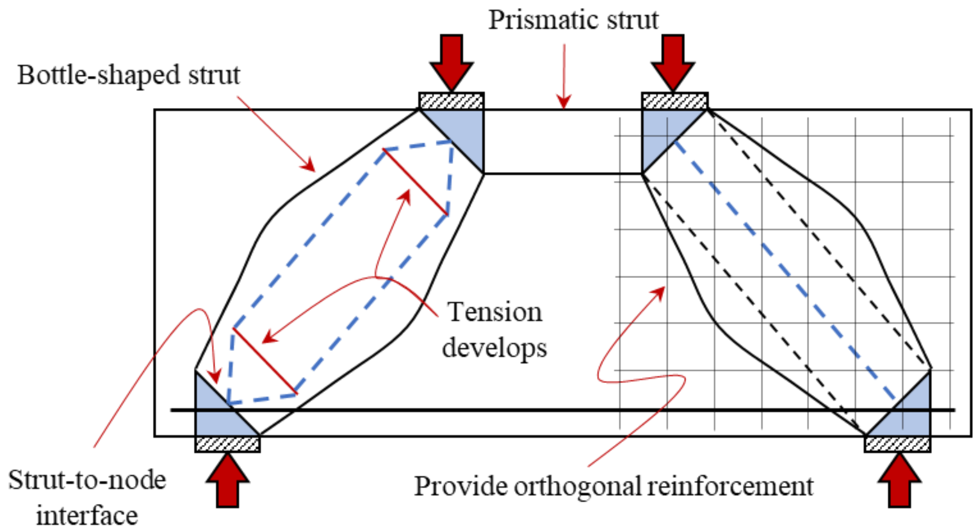
Forces are calculated in the truss model using conventional truss analysis methods (e.g., the method of joints) or structural analysis software. It is often beneficial to set up force calculations in the truss in a spreadsheet as it allows for easier recalculation of element forces if nodes change location.

Loads and load factors should be found based on AASHTO LRFD BDS Section 3. Distributed loads are generally applied at the nodes based on tributary areas around the nodes. More details on the applying bridge loads and load factors to a strut-and-tie model are provided in FHWA-NHI-17-071.

Additional details on struts, ties, and nodes are provided in the following sections.

Struts

Struts are often defined as either bottle-shaped or prismatic, as shown in Figure 12, depending on if compressive stresses can spread. Bottle-shaped struts are typically assumed when compressive stresses can spread, which results in tension developing in the strut as shown by the ties highlighted in Figure 12. Prismatic struts are assumed to occur when compressive stresses cannot spread either because a bordering tension region (as is the case in Figure 12) or geometric constraints. There are alternate strut definitions in ACI 318-19: interior and boundary struts. Interior struts have tension developing from bordering flexural stresses.



Source: FHWA.

Figure 12. Illustration. Typical definition of types of struts.

Regardless of the definition for struts, tension will develop in most struts, which is resisted by orthogonal reinforcement. This is prescribed in AASHTO LRFD BDS Article 5.8.2.6 as the minimum crack control reinforcement.

Vertical crack control reinforcement: $A_v / (b_w s_v) \geq 0.003$
AASHTO LRFD BDS Eqn. 5.8.2.6-1

Horizontal crack control reinforcement: $A_h / (b_w s_h) \geq 0.003$
AASHTO LRFD BDS Eqn. 5.8.2.6-2

The minimum crack control reinforcement helps to restrain the tension developing in struts and allows for redistribution of stresses in the member. This reinforcement has also been shown to limit crack widths in deep beam elements to less than 0.016 inches.

The compression capacity of struts is checked by the limiting compressive stress checks at nodes specified in AASHTO LRFD BDS Article 5.8.2.5.3.

Ties

Ties are the tension elements in a strut-and-tie model. The tension force in a tie, calculated based on the loading and strut-and-tie model, can be resisted by a combination of non-prestressed reinforcement, A_{st}, and prestressed reinforcement, A_{ps}. The strength of tension ties is defined in AASHTO LRFD BDS Article 5.8.2.4.

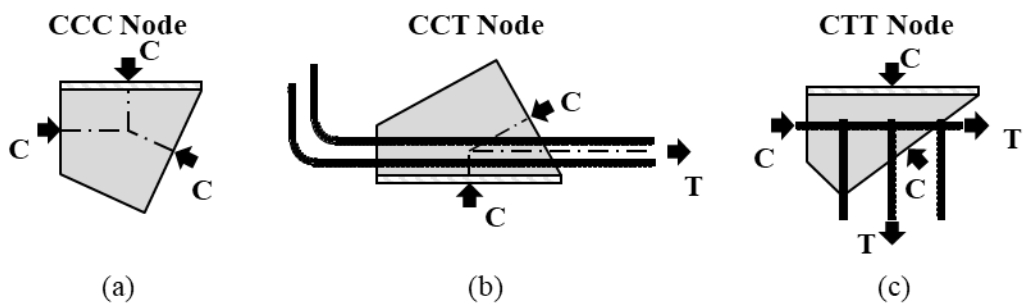
Tie Strength: $P_n = f_y A_{st} + A_{ps}(f_{pe} + f_y)$ AASHTO LRFD BDS Eqn. 5.8.2.4.1-1

where $(f_{pe} + f_y) \leq f_{py}$

The tie reinforcement must be properly anchored to transfer the tension force from the tie reinforcement to the node.

Nodes

There are three different types of nodes determined based on the forces intersecting the node. CCC nodes have compression forces in all intersecting elements, see Figure 13 (a). CCT nodes have one intersecting tension element with the rest compression, see Figure 13 (b). CTT nodes have two intersecting tension elements with the rest compression, see Figure 13 (c).



Source: FHWA.

Figure 13. Illustration. Typical node types in STM: (a) CCC, (b) CCT, and (c) CTT.

The strength of each node face needs to be calculated and compared to the demand from the respective intersecting element. The nominal resistance of a face of the node can be found as follows.

Strength of Node Face: $P_n = f_{cu} A_{cn}$ AASHTO LRFD BDS Eqn. 5.8.2.5.1-1

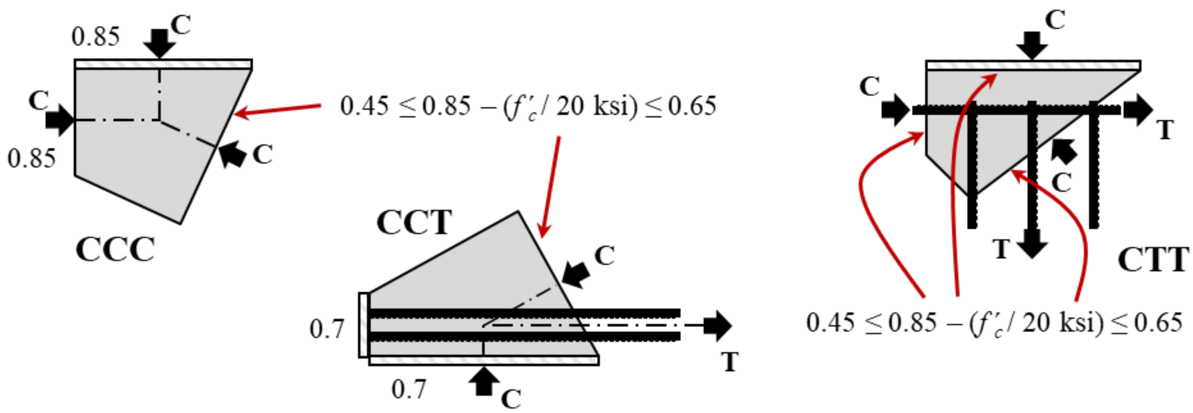
The limiting compressive stress at a node face is found based on AASHTO LRFD BDS Article 5.8.2.5.3.

Limiting compressive stress: $f_{cu} = m\, v\, f'_c$ AASHTO LRFD BDS Eqn. 5.8.2.5.3a-1

The concrete efficiency factor, v, depends on the type of node and the face of the node being investigated, as shown in Figure 14. The concrete efficiency factor dependent on the compressive strength has the following upper and lower limits.

Range for concrete efficiency factor dependent on concrete compressive strength:

$$0.45 \le v = 0.85 - (f'_c / 20 \text{ ksi}) \le 0.65$$



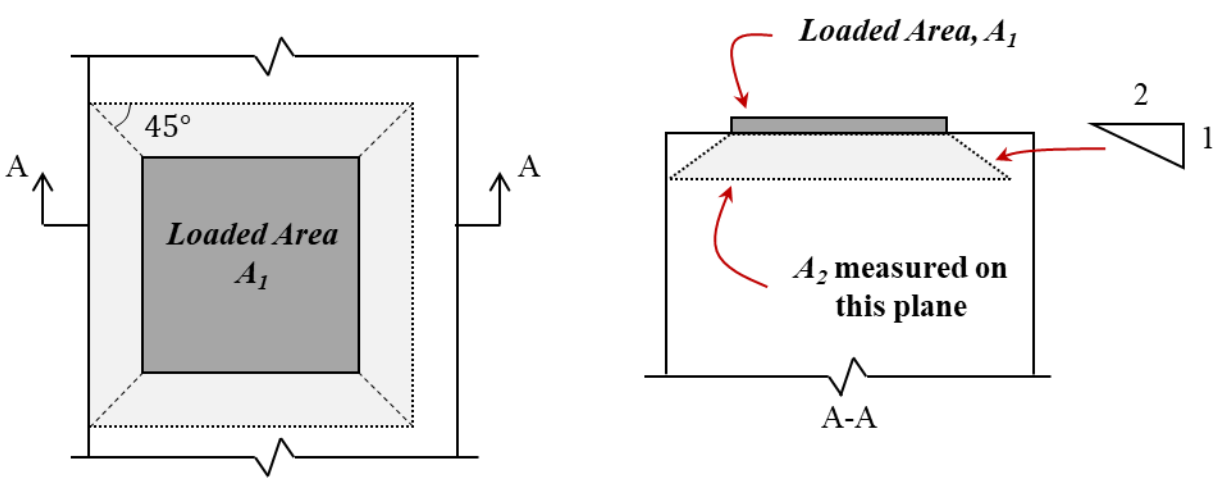
Source: FHWA.

Figure 14. Illustration. Concrete efficiency factors used for finding the limiting compressive stress in STM for conventional concrete.

The confinement modification factor, m, is based on the ratio of the area under the bearing device (A_1) and the area surrounding the bearing node (A_2).

Confinement modification factor: $m = \sqrt{(A_2 / A_1)}$ AASHTO LRFD BDS Article 5.8.2.5.3a

An illustration of the areas required for calculating the confinement modification factor is shown in Figure 15.

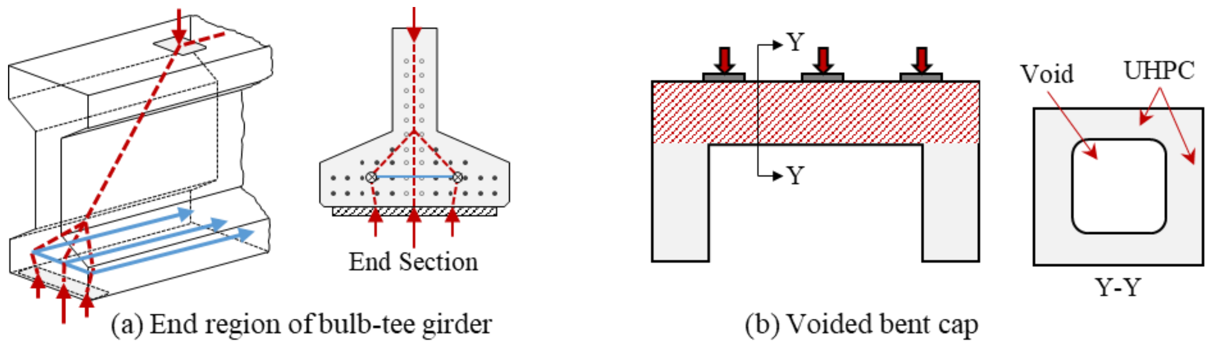


Source: FHWA.

Figure 15. Illustration. Areas needed for calculating the confinement modification factor.

CHAPTER 3. DESIGN CONSIDERATIONS FOR UHPC AND STM

The possible differences for using STM for UHPC structural elements may be grouped into two general categories: (1) effect of different materials properties and design philosophies for UHPC and (2) effect of different structural shapes and systems likely to be created using UHPC. The different material properties for UHPC were discussed in Chapter 2. UHPC members are likely to have voids or be used in combination with conventional concrete, which will create additional geometric discontinuities. Examples of two possible UHPC deep beam elements and regions are shown in Figure 16.



Source: FHWA.

Figure 16. Illustration. Examples of D-regions in UHPC structural elements, (a) end region of bulb-tee girder, based on NCHRP Research Report 994, and (b) voided bent cap.

The possible effect of these differences and possible options for modifying the current AASHTO LRFD BDS equations and procedures for STM are discussed in the following sections. These are broken down into the following categories.

- Implications on proportioning of tension ties (AASHTO LRFD BDS Article 5.8.2.4).
  - Strength of tension ties (based on UHPC Structural Design Guide Article 6.6.1).
  - Strain limitation for tension ties (UHPC Structural Design Guide Article 8).
  - Limitations for intersecting tension ties.
- Anchorage of tension tie reinforcement (UHPC Structural Design Guide Article 10.8).
- Proportioning of node regions (AASHTO LRFD BDS Article 5.8.2.5).
- Crack control reinforcement (AASHTO LRFD BDS Article 5.8.2.6).
- Elements with conventional concrete and UHPC.
- Additional design considerations.

There is currently no research on UHPC deep beam elements, so the recommendations provided in this section are only preliminary and should be investigated in more depth.

STRENGTH OF TENSION TIES (AASHTO LRFD BDS ARTICLE 5.8.2.4.1)

The strength of a tension tie in conventional concrete includes the strength of the non-prestressed and prestressed reinforcement as follows.

Strength of Tie: $P_n = f_y A_{st} + A_{ps} \times (f_{pe} + f_y)$ AASHTO LRFD BDS Eqn. 5.8.2.4.1-1

Where the summation of f_{pe} and f_y shall not be taken greater than the f_{py}.

AASHTO LRFD BDS Eqn. 5.8.2.4.1-1 assumes that the strain in the concrete at the centroid of the tension tie at failure of the tie is equal to the yield strain for the reinforcement (ε_y) or the prestressing (ε_{py}).

Yield strain for Grade 60 rebar: $\varepsilon_y = f_y / E_s = 0.00207$

Yield strain for low-relaxation strands (by definition): $\varepsilon_{py} = 0.010$

The strength of UHPC tension members is specified in UHPC Structural Design Guide Article 6.6.1 by the following equations.

Nominal resistance of tension members: $P_n = P_{UHPC} + P_s$

UHPC Structural Design Guide Eqn. 6.6.1-2

UHPC Component: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_g$ UHPC Structural Design Guide Eqn. 6.6.1-3

Steel Component: $P_s = 0.50E_s\gamma_u\varepsilon_{t,loc}A_s + A_{ps}[f_{pe} + 0.50E_s\gamma_u\varepsilon_{t,loc}]$

UHPC Structural Design Guide Eqn. 6.6.1-4

Where:

Limit on non-prestressed term: $(0.50E_s\gamma_u\varepsilon_{t,loc}) \leq 0.80f_y$

Limit on prestressed term: $[f_{pe} + 0.50E_s\gamma_u\varepsilon_{t,loc}] \leq 0.80f_{py}$

The resistance provided by a UHPC tie can be estimated using UHPC Structural Design Guide Eqn. 6.6.1-2 where A_g is replaced with A_{tie} for the UHPC component of the resistance.

Strength of tension tie in UHPC component: $P_n = P_{UHPC} + P_s$

Strength of UHPC in tension tie: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_{tie}$

UHPC Structural Design Guide Eqn. 1.6.6.1-3 introduces a reduction in the effective cracking stress to $0.60\gamma_u f_{t,cr}$ and ignores the post-cracking capacity.

The steel component of the tension tie strength, P_s, can be calculated using UHPC Structural Design Guide Eqn. 6.6.1-4 using the same limits for the non-prestressed and prestressed terms. UHPC Structural Design Guide Eqn. 6.6.1-4 introduce a strain limit for the tension ties, limiting the strain in the reinforcement in tension members to $0.50\gamma_u\varepsilon_{t,loc} \leq 0.80\varepsilon_y$ for non-prestressed reinforcement and a similar limit for prestressed reinforcement.

There is a minimum amount of reinforcement required for UHPC tension elements, as defined by UHPC Structural Design Guide Article 6.6.1.

Minimum reinforcement required in UHPC tension elements: $P_s \geq 0.80P_n$

This minimum reinforcement requirement should still be applied for tension ties. This limit is equivalent to setting an upper limit to the overall tie strength of $1.25P_s$.

Minimum reinforcement requirement in UHPC tension elements: $P_n \leq 1.25P_s$

This upper limit can be added to UHPC Structural Design Guide Eqn. 6.6.1-2, which results in the following design equation.

Strength of tension tie in UHPC component (with upper limit): $P_n = P_{UHPC} + P_s \leq 1.25P_s$

Utilizing the tension member provisions in UHPC Structural Design Guide Article 6.6 to design tension tie elements may be constraining as compared to a conventional concrete solution[1]. Further research on this topic is needed.

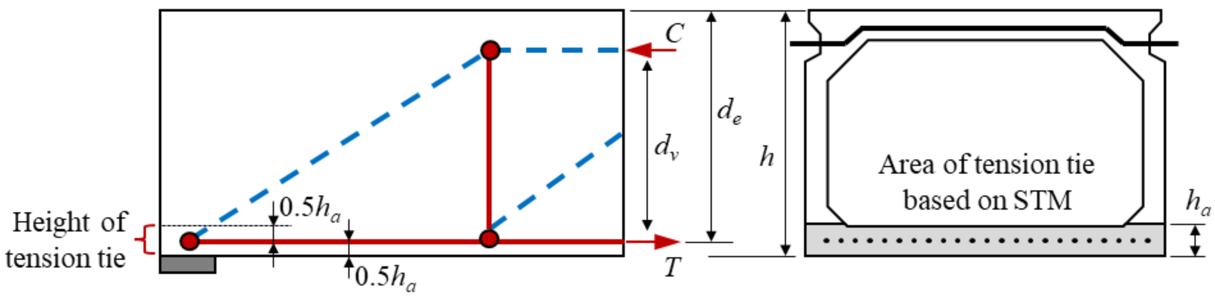
The area of the tension tie that will contribute to the strength provided by the tie will be dependent on the type of tie. There are several different types of ties in STM, some of these include:

- Longitudinal ties (like primary flexural reinforcement in typical simply supported elements),
- Vertical ties (like transverse reinforcement in typical simply supported elements),
- Hanger and ledge ties (used in inverted tee beams),
- Spreading ties (like those present when stresses from a post-tensioning anchor spread), and
- Corner ties (like those on the inside corner of a dapped end member).

AASHTO LRFD BDS Eqn. 5.8.2.4.1-1 is not dependent on the type of tension tie. There is guidance provided for proportioning these ties, e.g., the fan-shaped strut engagement of the transverse reinforcement shown in AASHTO LRFD BDS Figure C5.8.2.2-2. The guidance on proportioning ties can be reasonably used for determining the area of UHPC that will contribute to the strength of the tie (A_{tie}).

- **Longitudinal ties**: This area can be calculated using the area of the UHPC in the height of the tension tie (h_a), as shown in Figure 17.

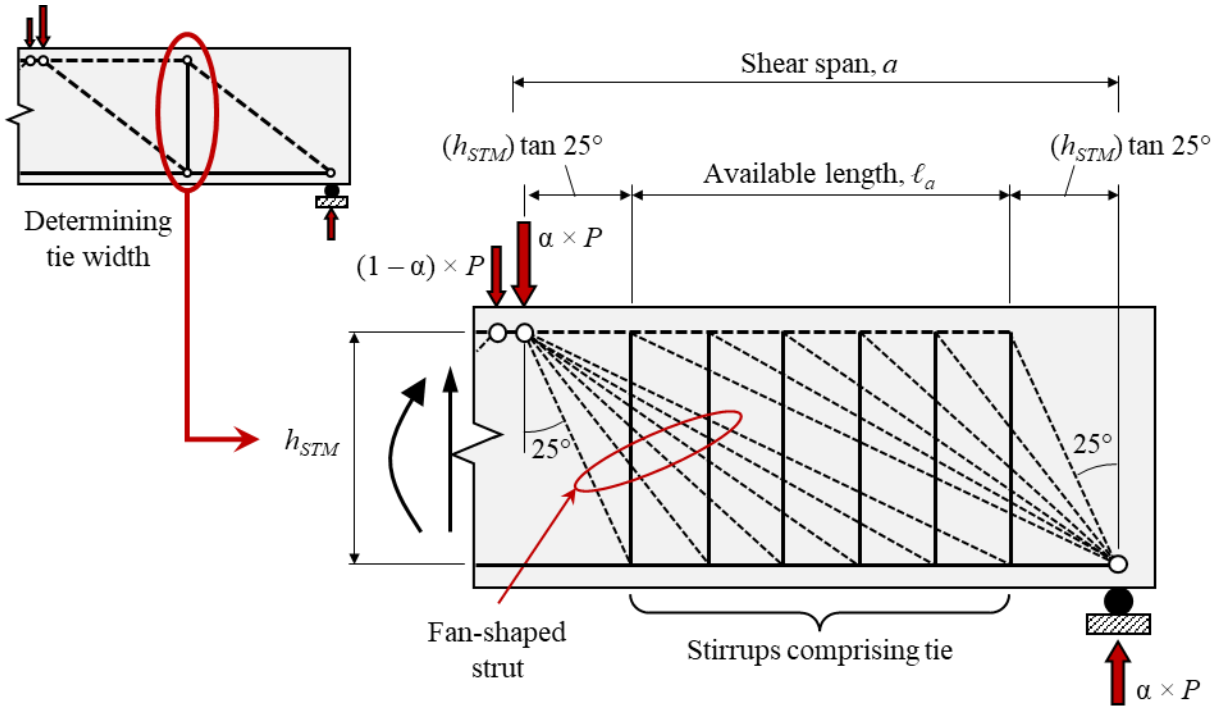
[1] The conventional concrete limit for a tie specified in AASHTO LRFD BDS Eqn. 5.8.2.4.1-1, if applied to a tie in a UHPC member, could result in the formation of a localization crack in the UHPC member prior to the member reaching its ultimate capacity. Within the UHPC Guide, localization is considered a tensile strain limit that is not to be exceeded. When engaging STM, a designer or owner who is considering relaxation of the stated strain-based restriction should carefully consider the potential implications on the performance of the specific structural element being designed.



Source: FHWA.

Figure 17. Illustration. Height of tension tie for UHPC box beam section.

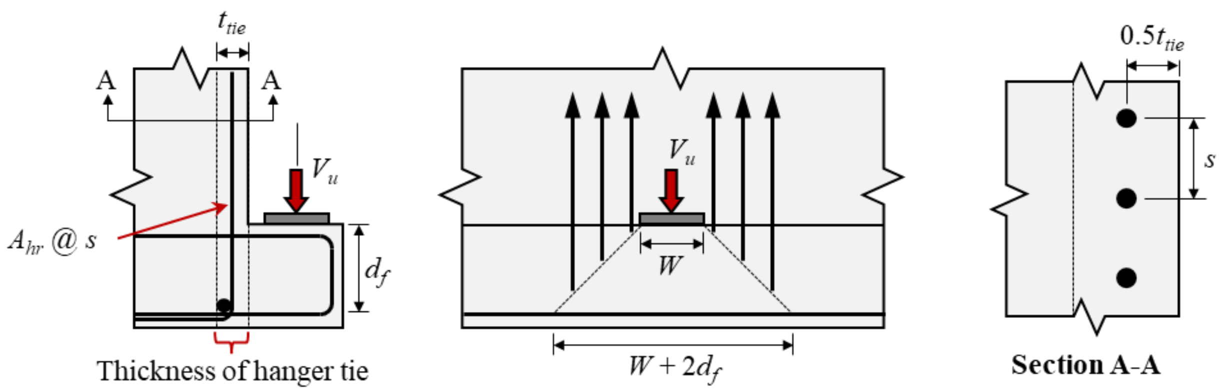
- **Vertical ties**: This area can be calculated using the area of the UHPC at the mid-height of the section within the available length (ℓ_a) defined by AASHTO LRFD BDS Figure C5.8.2.2-2 and shown in Figure 18.



Source: FHWA.

Figure 18. Illustration. Width of vertical tie, based on AASHTO LRFD BDS Figure C5.8.2.2-2.

- **Hanger ties**: This area can be calculated using the area of the UHPC within the (W + $2d_f$) region shown in AASHTO LRFD BDS Figure 5.8.4.3.5-2. The other dimension for the area can be calculated by taking two times the distance from the side face of the web to the center of the single leg of the vertical hanger tie. These dimensions are shown in Figure 19.

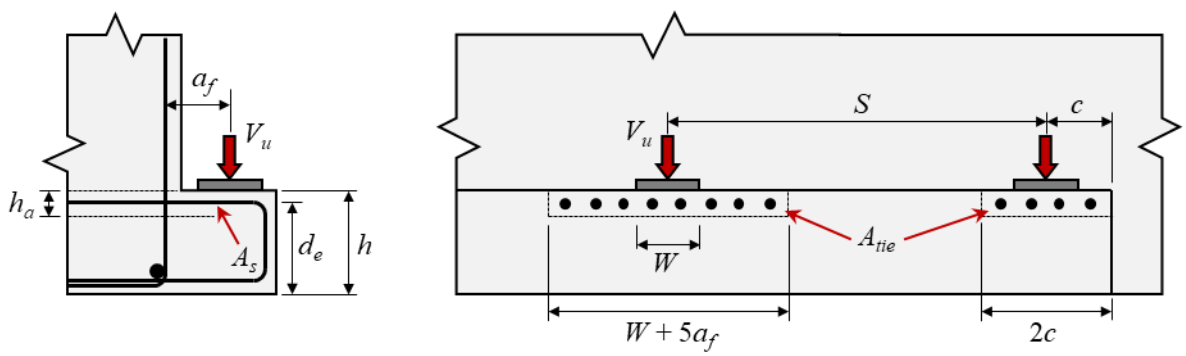


Source: FHWA.

Figure 19. Illustration. Dimensions needed for finding the area of UHPC for a hanger tie.

Area of hanger tie: $A_{tie} = t_{tie} \times (W + 2d_f)$

- **Ledge ties**: This area can be calculated using the area of the UHPC within the ($W + 5a_f$) or 2c region depending on if the load is located at the edge of the ledge, shown in AASHTO LRFD BDS Figure 5.8.4.3.3-1. The height of tie to use in this calculation can be found based on the centroid of the steel provided. These dimensions are shown in Figure 20.



Source: FHWA.

Figure 20. Illustration. Dimensions needed for finding the area of UHPC for a ledge tie.

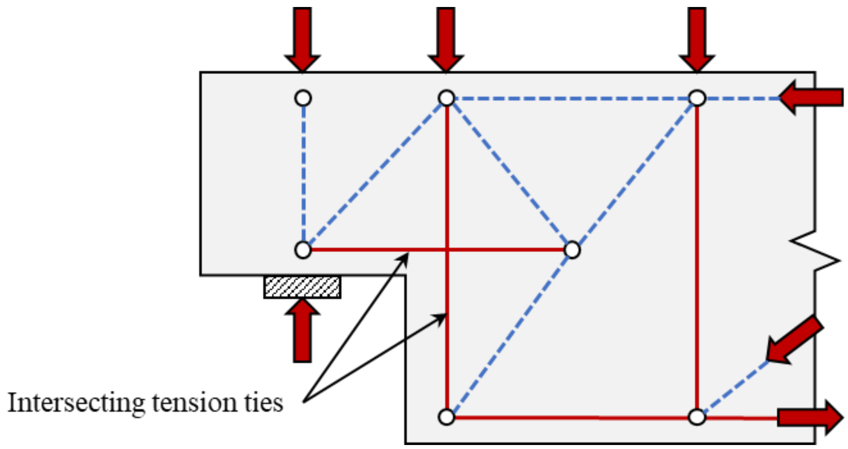
Height of ledge tie: $h_a = 2(h - d_e)$

- **Spreading ties**: The width of these ties is not always well defined. It is conservative to ignore the contribution of the UHPC for these ties. If the dimension of these ties can be determined based on reasonable engineering judgment, then the contribution may be included.
- **Corner ties**: The height or thickness of this tie can be found like hanger and ledge ties, two times the distance between the face of the corner and the center of the reinforcement. The width of this tie would typically be the width of the member, as long as the reinforcement is evenly distributed across the width of the member.

Sample calculations for a longitudinal tie and a vertical tie are provided in the deep beam example in the appendix.

INTERSECTING TENSION TIES

There are some instances in STM where two ties intersect. One of the primary examples of a member with intersecting tension ties is a dapped-end beam, as shown in Figure 21. In this model, two tension ties perpendicular to each other overlap near the corner of the dapped end. For conventional concrete members, the reinforcement for each tie is provided continuous through this intersection and properly anchored at the nodes that intersect with the compressive struts.



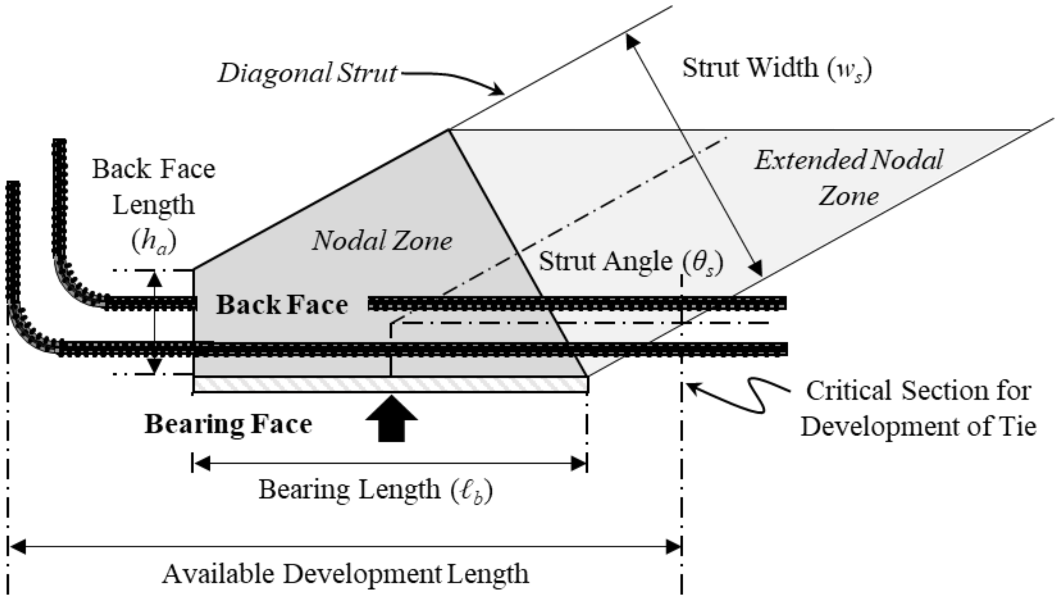
Source: FHWA.

Figure 21. Illustration. Typical strut-and-tie model for dapped end beams.

For UHPC members, the intersecting ties will create a state of biaxial tension. There is currently no research on how biaxial tension affects the material properties of UHPC, specifically the crack localization strain. Since the crack localization strain is used in calculating the tie strength using UHPC Structural Design Guide Eqn. 6.6.1-4, it is recommended to not use STM for designing members with intersecting ties. It may be conservative to simply ignore the P_{UHPC} term, but more research is needed.

ANCHORAGE OF TIE (AASHTO LRFD BDS ARTICLE 5.8.2.4.2)

The reinforcement in the tension tie must be properly developed. The available development length for a tie with a hook, which is typically required for conventional concrete members, is shown in Figure 22. The transfer and development length for a prestressing strand in UHPC can be found using UHPC Structural Design Guide Article 9.4.3. The development length for conventional reinforcement can be found using UHPC Structural Design Guide Article 10.8.



Source: FHWA.

Figure 22. Illustration. Details related to the available development length in a CCT node.

The required development lengths for reinforcement in UHPC will generally be significantly shorter than in conventional concrete. This may allow for straight bars to be used in place of the bent bars typically required for conventional concrete.

PROPORTIONING OF NODE REGIONS (AASHTO LRFD BDS ARTICLE 5.8.2.5)

As mentioned above, the strength of each node face needs to be calculated and compared to the demand from the respective intersecting element. The nominal resistance of a face of the node can be calculated as follows.

Strength of Node Face: $P_n = f_{cu} A_{cn}$ AASHTO LRFD BDS Eqn. 5.8.2.5.1-1

The limiting compressive stress at a node face is calculated based on AASHTO LRFD BDS Article 5.8.2.5.3.

Limiting compressive stress: $f_{cu} = m \, v \, f'_c$ AASHTO LRFD BDS Eqn. 5.8.2.5.3a-1

The concrete efficiency factor, v, depends on the type of node, inclusion of crack control reinforcement per AASHTO LRFD BDS Article 5.8.2.6, and the face of the node being investigated, as shown in Figure 14.

Concrete efficiency factors were developed by Birrcher et al. (2009) based on large-scale experimental testing and database analysis for conventional concrete deep beams. The efficiency factor for the strut-to-node interface was made dependent on the concrete compressive strength as higher strength conventional concrete mixtures lead to lower efficiency of the concrete on this interface. This is because shear cracks begin to form through coarse aggregate for high strength concrete.

The strut-to-node interface strength and CCT node strengths may be better represented by research related to UHPC shear (with biaxial tension and compression). The UHPC Structural Design Guide Article C7.3.3 states:

> *For a more detailed analysis, the crushing limit in a cracked UHPC member subjected to shear forces can be checked by ensuring that the stress in the compression strut, f_2, is less than half of the compression strength, $f_2 \leq 0.5f'_c$.*

Based on this guidance, an efficiency factor of 0.5 may be used for strut-to-node interfaces and CTT node bearing and back faces. The recommended efficiency factors for UHPC are summarized in Table 1. The efficiency factors for the bearing and back faces of nodes for CCC and CCT of conventional concrete members are likely still appropriate for UHPC members.

Table 1. Recommended efficiency factors for UHPC nodes.

Face	CCC Node	CCT Node	CTT Node
Bearing Face	0.85	0.70	0.50
Back Face	0.85	0.70	0.50
Strut-to-Node Interface	0.50	0.50	0.50

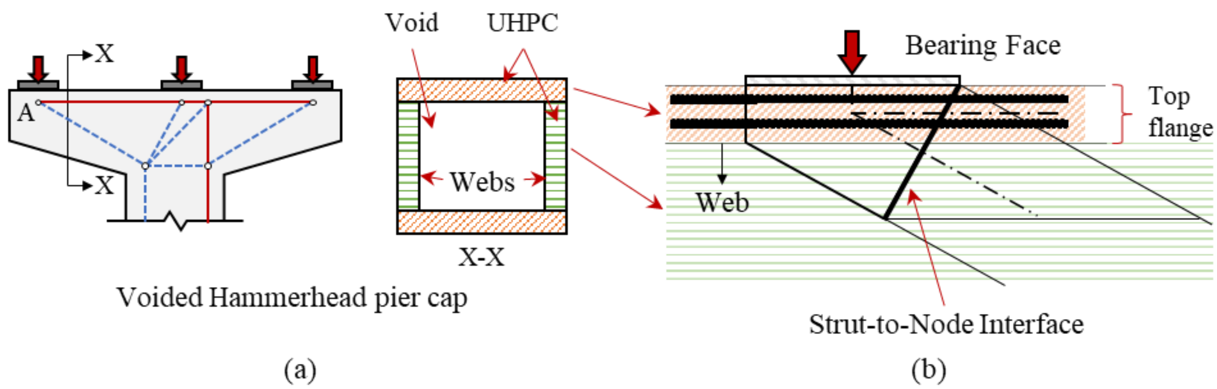
As discussed below, the minimum crack control reinforcement is likely provided by the steel fibers, so the efficiency factor of 0.45 typically used for conventional concrete when minimum crack control reinforcement is not present need not apply for UHPC components.

The confinement modification factor was also developed by Birrcher et al. (2009) based on results from conventional concrete deep beams. Until more research is performed on the effects of triaxial passive confinement on UHPC, the confinement modification factor is suggested to be assumed equal to 1.0 for UHPC.

Recommended confinement modification factor for UHPC: $m = 1.0$

The area of the faces in a UHPC node can be calculated like those in a conventional concrete member.

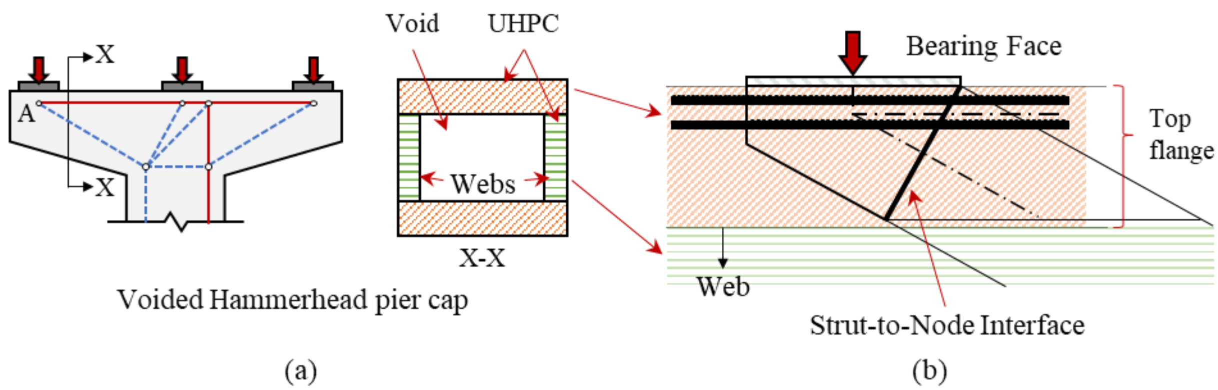
The presence of voids should be considered when calculating areas, and designers are encouraged to place voids outside of nodal zones. An illustration of a hypothetical voided hammerhead pier cap is shown in Figure 23; this is not an actual recommended design that has been vetted, but it is realistic that UHPC substructure elements would have voids to reduce weight and take advantage of the enhanced properties of UHPC. A typical strut-and-tie model is shown on the elevation view of the pier cap in Figure 23 (a). In the model and member shown in Figure 23 (b), the void extends into Node A. The area of the strut-to-node interface would need to be reduced by the area of the void.



Source: FHWA.

Figure 23. Illustration. (a) Elevation with strut-and-tie model and section view of a voided UHPC hammerhead pier cap with thin top and bottom flanges and (b) strut-to-node interface at Node A.

The top and bottom flanges could be increased, so that the void would not extend into the nodal zones, as shown in Figure 24, but this will not eliminate the effect of the void on the behavior of the member.

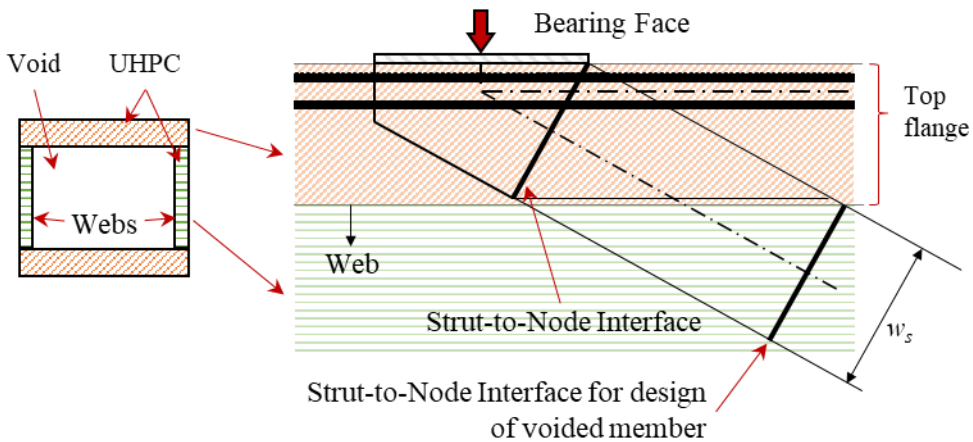


Source: FHWA.

Figure 24. Illustration. (a) Elevation with strut-and-tie model and section view of a voided UHPC hammerhead pier cap with thicker top and bottom flanges and (b) strut-to-node interface at Node A.

Typically, stresses are checked only at the node faces, including the strut-to-node interface. The stresses in the adjacent struts are assumed to spread in the web away from the node, so the strut-to-node interface is assumed to be the critical section. If a void is present, it may shift the critical failure plane away from the strut-to-node interface into the strut.

Until further research is conducted, the presence of a void should be conservatively factored into the design for the strut-to-node interface. The cross-sectional area of struts crossing voids should be set equal to the minimum cross-sectional area of the UHPC along the length of the strut, shown in Figure 25.



Source: FHWA.

Figure 25. Illustration. Conservative strut-to-node interface plane to use for design of voided members.

Sample calculations for CCT and CCC nodes in a voided UHPC deep beam member are provided in the example in the appendix.

CRACK CONTROL REINFORCEMENT (AASHTO LRFD BDS ARTICLE 5.8.2.6)

Crack control reinforcement is intended to limit the width of cracks and allow for the redistribution of internal stresses, which is an important factor for STM. Members without the minimum crack control reinforcement (specified by AASHTO LRFD BDS Article 5.8.2.6) need to use a concrete efficiency factor of 0.45, which is lower than if crack control reinforcement is provided for most cases.

For conventional concrete, crack control reinforcement is specified in the vertical and horizontal directions as follows:

Limit for Vertical Reinforcement: $A_v / (b_w s_v) \geq 0.003$

AASHTO LRFD BDS Eqn. 5.8.2.6-1

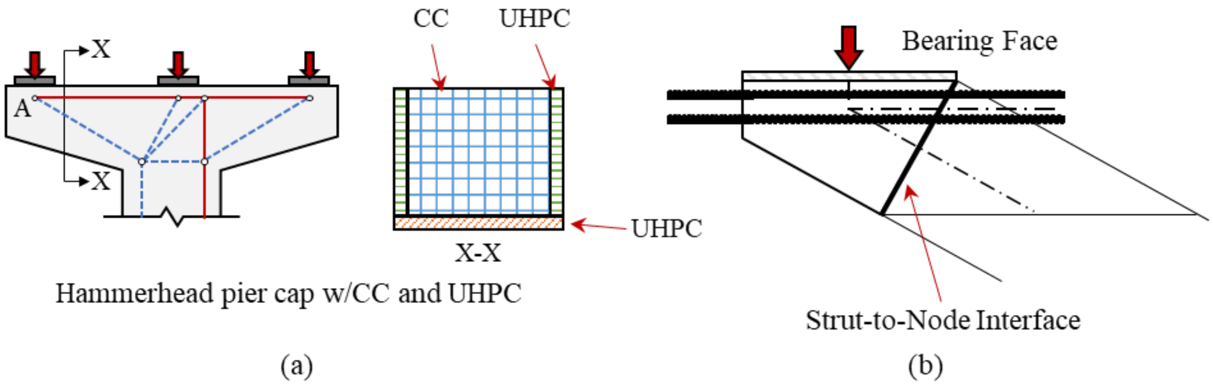
Limit for Horizontal Reinforcement: $A_h / (b_w s_h) \geq 0.003$

AASHTO LRFD BDS Eqn. 5.8.2.6-1

These limits are intended to provide 0.3 percent reinforcement perpendicular to the strut. UHPC commonly has approximately 2.0 percent by volume of steel fiber reinforcement. While fiber orientation will depend on many different factors (e.g., casting, presence of discrete reinforcement), a UHPC mixture with 2.0 percent steel fibers can be assumed to meet minimum crack control reinforcement requirements without any additional reinforcement. This is also consistent with the findings related to UHPC subjected to beam shear; the UHPC inherently satisfies the required minimum transverse reinforcement required for conventional concrete.

ELEMENTS WITH CONVENTIONAL CONCRETE AND UHPC

There have been proposed substructure elements with a combination of UHPC, typically on the outside, and conventional concrete. A sample of a hybrid hammerhead pier cap element with UHPC on the outside and conventional concrete on the inside is shown in Figure 26.



Source: FHWA.

Figure 26. Illustration. (a) Hammerhead pier cap with conventional concrete and UHPC and (b) node geometry for Node A.

The specifications for interface shear transfer in UHPC Structural Design Guide Article 7.4 should be followed to ensure composite behavior between the UHPC and conventional concrete portions of the member. If there is insufficient shear interface capacity between the conventional concrete and UHPC to create composite action, then the UHPC should be ignored when calculating the node and tie strengths, assuming that the primary tension steel is in the conventional concrete and that the concentrated loads are applied to the conventional concrete. This type of member should be designed using the strut-and-tie method for conventional concrete specified in AASHTO LRFD BDS Article 5.8. All loads in this case would be carried through the conventional concrete.

If there is composite action between the UHPC and conventional concrete, then the UHPC can likely be included in the resistance calculations. The contribution of the UHPC likely needs to be reduced or eliminated as the conventional concrete and UHPC reach their ultimate strength at different strains.

Conventional concrete will reach its ultimate compressive strength at strains between 0.00190 and 0.00222 (for concrete strengths between 4 and 10 ksi assuming the stress-strain relationship by Thorenfeldt, Tomasqewicz, and Jensen (1987) presented in Collins and Mitchell (1991)). The approximate UHPC stresses at these strains for different compressive strength UHPC are summarized in Table 2, with the ratio of this stress to the UHPC strength also provided. This ratio decreases as the UHPC compressive strength increases with the lowest ratio at approximately 0.5 for a UHPC with a 30 ksi compressive strength.

Table 2. UHPC stress at ultimate strain for CC for different UHPC compressive strengths.

f'_c (ksi)	E_c (ksi)	$f_{c@0.0019}$ (ksi)	ratio	$f_{c@0.0023}$ (ksi)	ratio
17.5	6,429	12.22	0.70	14.79	0.84
20.0	6,719	12.77	0.64	15.45	0.77
22.5	6,985	13.27	0.59	16.07	0.71
25.0	7,232	13.74	0.55	16.63	0.67
27.5	7,463	14.18	0.52	17.17	0.62
30.0	7,681	14.59	0.49	17.67	0.59

These assumed strains at ultimate strength are less than the 0.003 assumed in AASHTO LRFD BDS (see Article 5.6.4.4). A lower assumed strain at ultimate will lead to a more conservative estimated capacity until more research can be performed.

The strength of a node face can be found by adding the conventional concrete component to a reduced UHPC component. The proposed strength of a hybrid conventional concrete and UHPC node is as follows.

Strength of Hybrid Node Face: $P_n = f_{cu,CC} A_{cn,CC} + 0.5 f_{cu,UHPC} A_{cn,UHPC}$

The crack control reinforcement for the conventional concrete portion of the member should be provided as specified in AASHTO LRFD BDS until further research can be performed.

Reinforcement for tension ties can be provided in either the UHPC or conventional concrete as long as there is sufficient shear transfer provided between the two materials along the length of the tie or concentrated in nodal regions.

ADDITIONAL DESIGN CONSIDERATIONS

There are several additional important design considerations not explicitly considered in the design process for STM that may be critical for D-regions in UHPC components. These additional design considerations include the following.

- **Sharp Corners in Voids or Other Geometric Discontinuities**: Sharp corners should generally be avoided in voids or other geometric discontinuities in UHPC components. A chamfer at the corners should be used to help avoid stress concentrations and restrained shrinkage cracks that can happen at a sharp corner.
- **Direct Load Path from Concentrated Loads into Body of Member**: The location and size of the load and support bearings and the portions of the member into which stresses flow should ensure a direct load path is available. If there is not a direct load path, then a three-dimensional strut-and-tie model should be used for the design.
- **Torsional Resistance**: The torsional resistance of a section should be considered if there may be an unbalanced load.
- **Stability of Member and Section**: As compared to conventional concrete members, UHPC members may be more slender and may have significantly thinner webs and flanges. Some UHPC designs may begin to resemble traditional steel members.

Consideration should be given to including diaphragms, stiffeners, or other means to ensure that the section and member remain locally and globally stable under all load cases.

CHAPTER 4. SUMMARY AND CONCLUSIONS

The current UHPC Structural Design Guide provides limited guidance on the use of STM with UHPC members, leaving it up to the designer and the owner to determine appropriate design criteria. The initial framework provided in this report are a starting point for discussions and future research related to the use of STM for UHPC members.

CHAPTER 5. REFERENCES

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APPENDIX. SIMPLY-SUPPORTED DEEP BEAM EXAMPLE

INTRODUCTION

The use of the possible framework introduced in this report is demonstrated in the example provided in this appendix. This example is a basic simply-supported UHPC deep beam design based on the conventional concrete deep beam Design Example 1 in FHWA-NHI-17-071.

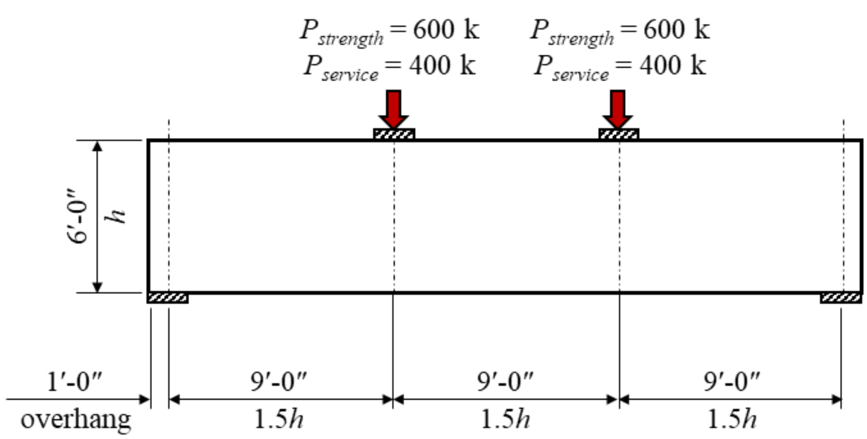
The steps in the STM design process for conventional concrete deep beams include the following.

- Design Step 1: Define strut-and-tie model inputs.
- Design Step 2: Determine the locations of the B- and D-regions.
- Design Step 3: Define load cases.
- Design Step 4: Analyze structural components.
- Design Step 5: Size structural components using the shear serviceability check.
- Design Step 6: Develop a strut-and-tie model.
- Design Step 7: Proportion ties.
- Design Step 8: Perform nodal strength checks.
- Design Step 9: Proportion crack control reinforcement.
- Design Step 10: Provide necessary anchorage for ties.
- Design Step 11: Draw reinforcement layout.

The same basic steps will be used for the UHPC deep beam in this example.

DESIGN STEP 1: DEFINE STRUT-AND-TIE MODEL INPUTS

The outside beam geometry used in this example will be the same as Design Example 1 from FHWA-NHI-17-071, shown in Figure 27.



Source: FHWA.

Figure 27. Illustration. Beam defined for Design Example 1, based on FHWA-NHI-17-071.

The deep beam has a height, h, of 6 feet and total length, L, of 27 feet with equal point loads at the third points. The point loads are specified to have a strength demand, $P_{strength}$, of 600 kips and service demand, $P_{service}$, of 400 kips. The width of the beam, b, is assumed to be 4 feet.

The UHPC material properties are as follows.

- Compressive strength for use in design and analyses: f'_c = 17.5 ksi
- Correction factor for modulus of elasticity: K_1 = 1.0
- UHPC unit weight (including reinforcement): w_c = 0.160 kcf
- Reduction factor for compression: α_u = 0.85
- Ultimate compression strain: ε_{cu} = 0.0035
- Effective cracking strength: $f_{t,cr}$ = 1.0 ksi
- Effective cracking strength at transfer: $f_{t,cri} = 0.75 f_{t,cr}$ = 0.75 ksi
- Crack localization strength: $f_{t,loc}$ = 1.0 ksi
- Crack localization strain: $\varepsilon_{t,loc}$ = 0.005
- Reduction factor for tension: γ_u = 1.0

The modulus of elasticity of the concrete for use in design is calculated using UHPC Structural Design Guide Eqn. 4.2.3-1.

Modulus of elasticity for use in design: $E_c = 2500 K_1 f'^{0.33}_c$ = 6,429 ksi

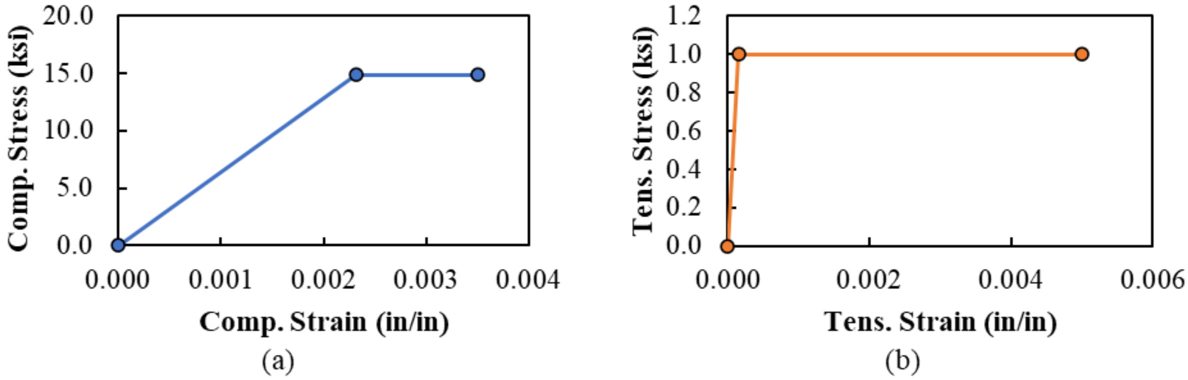
The elastic compression strain is calculated using UHPC Structural Design Guide Eqn. 1.4.2.4.2-1 assuming a linear elastic response up to the reduced compression strength.

Elastic compression strain: $\varepsilon_{cp} = (\alpha_u f'_c) / E_c$ = 0.00231

The cracking strain is calculated assuming a linear elastic response up to the effective cracking strength.

Effective cracking strain: $\varepsilon_{t,cr} = (\gamma_u f_{t,cr}) / E_c$ = 0.000156

The stress-strain relationships used for design are shown in Figure 28. An elastic-plastic response is assumed for the UHPC in both compression and tension.



Source: FHWA.

Figure 28. Graphs. UHPC stress versus strain responses for (a) compression and (b) tension.

The material properties for the conventional steel reinforcement (Grade 60) are as follows:

- Modulus of elasticity: E_s = 29,000 ksi
- Yield strength: f_{sy} = 60 ksi

DESIGN STEP 2: DETERMINE THE LOCATIONS OF B- AND D-REGIONS

The entire beam in this example is a deep beam. D-regions extend a distance, d, from any supports or point loads. The exact location of the primary longitudinal tension reinforcement is not known at this point. Typically, the centroid of the reinforcement will not be more than 0.1h from the bottom, which would result in all parts of the beam being less than d = 0.9h away from a support or point load.

DESIGN STEP 3: DEFINE LOAD CASES

The load cases are defined by FHWA-NHI-17-071 as the following.

- $P_{service}$ = 400 kips (per point load) for Service I Load Combination
- $P_{strenght}$ = 600 kips (per point load) for Strength I Load Combination

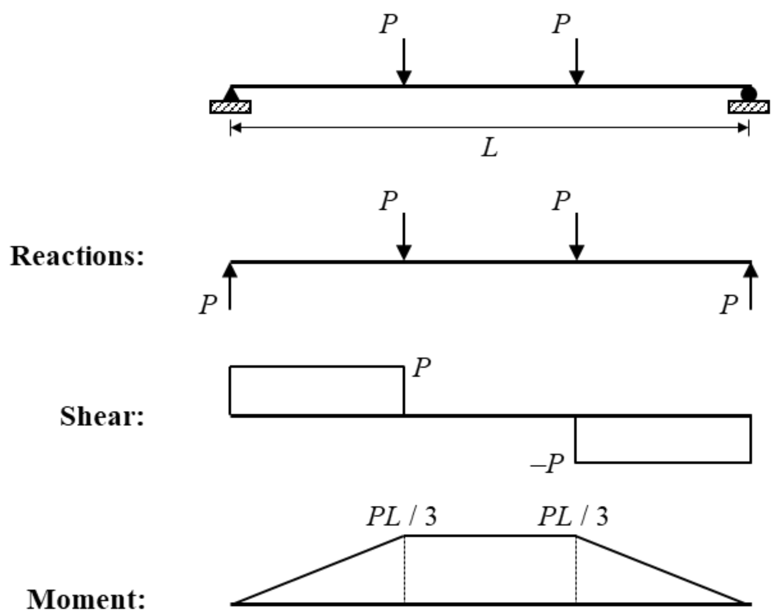
The point loads are applied to bearings with a given dimension of 12 inches long by 48 inches wide. The self-weight of the beam is assumed to already be included in these load cases.

DESIGN STEP 4: ANALYZE STRUCTURAL COMPONENTS

The reactions and shear and moment diagrams are calculated as shown in Figure 29 and summarized below.

- Left reaction: $R_{L,service}$ = 400 kips; $R_{L,strength}$ = 600 kips

- Right reaction: $R_{R,service}$ = 400 kips; $R_{R,strength}$ = 600 kips
- Max. shear: $V_{max,service}$ = 400 kips; $V_{max,strength}$ = 600 kips
- Max moment: $M_{max,service}$ = 3,600 kip-ft; $M_{max,strength}$ = 5,400 kip-ft



Source: FHWA.

Figure 29. Illustration. Reactions, shear diagram, and moment diagram for beam in the example.

The moment and shear demand are often used for conventional concrete members to aid in the initial sizing of the component.

DESIGN STEP 5: SIZE STRUCTURAL COMPONENTS USING SHEAR SERVICEABILITY CHECK

A conventional concrete deep beam is often initially sized based on the estimated cracking strength for the deep beam calculated using AASHTO LRFD BDS Eqn. C5.8.2.2-1.

Estimated cracking resistance for conventional concrete deep beams:

$$V_{cr} = [0.2 - 0.1(a / d)]\sqrt{f'_c}\, b_w\, d \qquad \text{AASHTO LRFD BDS Eqn. C5.8.2.2-1}$$

$$\text{where } 0.0632 \sqrt{f'_c}\, b_w\, d \leq V_{cr} \leq 0.158 \sqrt{f'_c}\, b_w\, d$$

The cracking of a UHPC member for this step can be reasonably estimated using UHPC Structural Design Guide Eqn. 7.3.3-3 with the effective cracking strength, $f_{t,cr}$, used instead of the effective crack localization strength, $f_{t,loc}$, and assuming no transverse reinforcement when calculating the principal angle direction.

UHPC contribution to shear resistance: $V_{UHPC} = \gamma_u f_{t,loc} b_v d_v \cot\theta$

UHPC Structural Design Guide Eqn. 7.3.3-3

Assumed cracking resistance of UHPC in shear: $V_{UHPC,cr} = \gamma_u f_{t,cr} b_v d_v \cot \theta$

Given that this step is a design approximation that will be modified later as needed, the maximum principal angle for the effective crack localization strain from UHPC Structural Design Guide Table B2-1 can conservatively be used.

- Effective crack localization strain: $\gamma_u \varepsilon_{t,loc} = 0.005$
- Maximum principal angle (from UHPC Structural Design Guide Table B2-1): 47.5°

The area of the longitudinal steel should also be estimated at this stage. This area will initially be estimated assuming a rectangular cross section and using UHPC Structural Design Guide Article 6.3.

The depth of the steel will be based on the cover, diameter of transverse reinforcement, size of longitudinal bars, and number of layers of longitudinal bars. The initially assumed values for these will be as follows.

- Cover for UHPC (UHPC Structural Design Guide Article 10.1): c_c = maximum of $1.5\ell_{fiber}$ and 0.75 inch = 1.5(0.5 inch) = 0.75 inch
- Transverse reinforcement: No. 5 bars with d_b = 0.625 inch
- Longitudinal bars: Two layers of No. 10 bars, d_b = 1.27 inch
- Clear spacing between layers of bars (UHPC Structural Design Guide Article 10.3): s_c = maximum of $1.5\ell_{fiber}$ and 0.75 inch = 1.5(0.5 inch) = 0.75 inch

Based on these assumptions, the distance from the top of the beam to the centroid of the longitudinal reinforcement is estimated as follows.

Reinforcement depth: d = 72 inch – 0.75 inch – 0.625 inch – 1.27 inch – 0.5(0.75 inch) = 69.0 inch

The lower-bound limit for the effective shear depth will be used in this example.

Effective shear depth: d_v = maximum of ($0.9d_e$ = 62.1 inch) and ($0.72h$ = 51.8 inch) = 62.1 inch

The assumed cracking resistance can be used to approximate an initial web width to estimate an initial size of an internal void.

Assumed cracking resistance of UHPC in shear: $V_{UHPC} = \gamma_u f_{t,cr} b_v d_v \cot \theta = V_{max,service}$

V_{UHPC} = (1.0)(1 ksi)(b_v)(62.1 inch)cot(47.5°) = 400 kips

Solving for b_v: b_v = 7.03 inch

A cross section with three 3-inch webs will be initially investigated. The bottom flange size will be based on the required cover for the two layers of No. 10 bars.

Required bottom flange thickness: $t_{bf,req}$ = 2(0.75 inch) + 0.625 inch + 2(1.27 inch) + (0.75 inch) = 5.415 inch

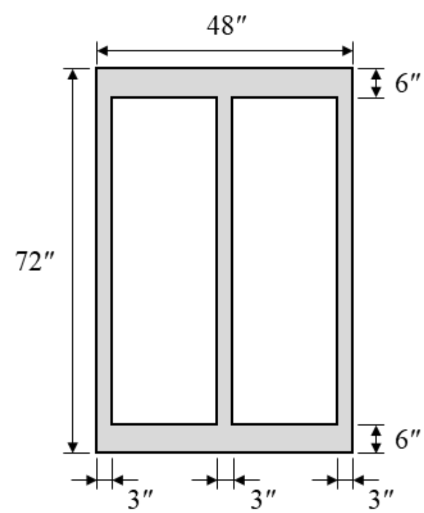
This value will be rounded up to the nearest inch. This thickness will also allow for the UHPC tension tie area to be entirely in the bottom flange.

Initial estimate for bottom flange thickness: t_{bf} = 6 inch

The initial top flange thickness will be assumed to be equal to the bottom flange thickness.

Initial estimate for top flange thickness: $t_{tf} = t_{bf}$ = 6 inch

This will lead to the following section initially considered in the design.



Source: FHWA.

Figure 30. Illustration. Initial cross section for design of UHPC deep beam in the example.

The section geometry will be modified as needed during the STM design process.

DESIGN STEP 6: DEVELOP A STRUT-AND-TIE MODEL

The strut-and-tie model for this example will be assumed to be the same as that used in FHWA-NHI-17-071, where both a single panel and two panel model are assumed.

The centroid of the top line of nodes is dependent on the centroid of the compression force resultant from a flexure analysis. Two layers of non-prestressed reinforcement will be used and designed such that $\phi M_n \geq M_u$ = 64,800 kip-inch. The centroid of each layer of reinforcement is based on the cover and spacing requirements (0.75 inch), diameter of No. 5 transverse reinforcement assumed (0.625 inch), and diameter of the No. 10 bars (1.27 inch). A summary of the reinforcement required for flexure is provided in Table 3. The reinforcement was kept symmetrical between the two layers to simplify the centroid of the tension tie.

Table 3. Area of reinforcement needed for flexural design.

Layer #	A_s	y_s
1	(6) No. 10 = 7.62 inch^2	2.0 inch
2	(6) No. 10 = 7.62 inch^2	4.0 inch

The design with the reinforcement in Table 3 and section shown in Figure 30 would result in the following properties.

- Factored nominal flexural resistance: ϕM_n = 82,357 kip-inch
- Centroid of compression resultant (from top): y_{Cc} = 3.1 inch

The centroid of the compression resultant is approximately equal to the centroid of the top flange in this example. A constant stress is assumed over the depth of the compressive stress region, so it is reasonable to assume the centroid of the top line of nodes is equal to the centroid of the top flange. The top flange thickness can be modified if the flexural analysis results in a centroid of the compression resultant that is significantly different than the centroid of the top flange.

- Centroid of top nodes (from top, for STM): y_{Cc} = 3.0 inch

The centroid of the bottom lines of nodes will be assumed to be at the centroid of the tension reinforcement.

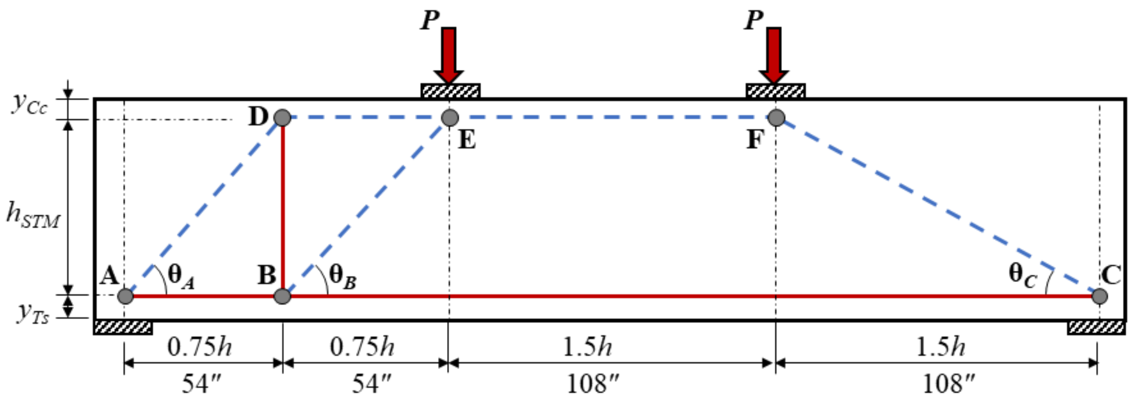
- Centroid of tension reinforcement (from bottom): y_{Ts} = 3.0 inch

The distance between the top and bottom line of nodes can be calculated as follows.

Distance between top and bottom nodes: $h_{STM} = h - y_{Cc} - y_{Ts}$

$$h_{STM} = 72 \text{ inch} - 3 \text{ inch} - 3 \text{ inch} = 66 \text{ inch}$$

The strut-and-tie model for this deep beam is shown in Figure 31.



Source: FHWA.

Figure 31. Illustration. Strut-and-tie model used for the example.

The strut angles can be calculated as follows.

- Strut AD angle: $\theta_A = \tan^{-1}(66 \text{ inch} / 54 \text{ inch}) = 50.7°$
- Strut BE angle: $\theta_B = \theta_A = 50.7°$
- Strut CF angle: $\theta_C = \tan^{-1}(66 \text{ inch} / 108 \text{ inch}) = 31.4°$

All strut angles are greater than the minimum strut angle of 25° specified in AASHTO LRFD BDS Article C5.8.2.2.

The truss forces are calculated using the method of joints and are summarized in Table 4.

Table 4. Summary of element forces for initially assumed STM model.

Element	**Force (kips)**	**Element Type**
AB	490.9	Tie
AD	775.2	Strut
BC	981.8	Tie
BD	600.0	Tie
BE	775.2	Strut
CF	1150.6	Strut
DE	490.9	Strut
EF	981.8	Strut

These strut forces are used to proportion the ties (Design Step 7) and perform the nodal strength checks (Design Step 8).

DESIGN STEP 7: PROPORTION TIES

There are three ties that need to be proportioned in this model: AB, BC, and BD. These ties will be designed using UHPC Structural Design Guide Article 6.6.1 with the recommended procedure for calculating the UHPC area in the tension tie.

Tie AB and Tie BC (Longitudinal Ties)

The area of the longitudinal tension ties (Tie AB and BC) is equal to the UHPC area contained in the height of the tension tie, h_a, which is based on the centroid of the longitudinal reinforcement.

Distance from bottom of section to centroid of tension tie (2 layers):

$$0.5h_a = \text{cover} + d_{b,transverse} + d_{b,longitudinal} + 0.5s_b$$

$$0.5h_a = 0.75 \text{ inch} + 0.625 \text{ inch} + 1.27 \text{ inch} + 0.5(0.75 \text{ inch}) = 3 \text{ inch}$$

$$h_a = 6 \text{ inch}$$

This is the same height as the bottom flange (by design), so the tie area is this height times the width of the beam.

Initial estimated tie area for Tie AB and Tie BC: $A_{tie,AB} = A_{tie,BC} = h_a \times b_{bf}$

$$A_{tie,AB} = A_{tie,BC} = (6\text{ inch})(48\text{ inch}) = 288\text{ inch}^2$$

The area of steel in the tension tie should be subtracted from the area of the UHPC in the tension tie. This area will be neglected at this stage of the design process.

The strength of the tie consists of a UHPC component and steel components as calculated using the following equations.

Strength of UHPC in tension tie: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_{tie}$

$$P_{UHPC} = (0.60)(1.0)(1.0\text{ ksi})(288\text{ inch}^2) = 172.8\text{ kips}$$

Limit on non-prestressed term: $(0.50 E_s \gamma_u \varepsilon_{t,loc}) \leq 0.80 f_y$

$$(0.50)(29{,}000\text{ ksi})(1.0)(0.005) = 72.5\text{ ksi} \leq 0.80(60\text{ ksi}) = 48\text{ ksi}$$

The upper limit on the stress in the non-prestressed reinforcement, $0.8f_y$, controls for this example with the effective localization strain of 0.005.

Steel Component: $P_s = 0.50 E_s \gamma_u \varepsilon_{t,loc} A_s + A_{ps}[f_{pe} + 0.50 E_s \gamma_u \varepsilon_{t,loc}]$

UHPC Structural Design Guide Eqn. 6.6.1-4

$$P_s = (48\text{ ksi})(A_s)$$

The amount of reinforcement will be designed using these equations (setting capacity equal to demand), so A_s will be left as a variable at this stage.

The total strength of the tie is the summation of the UHPC and steel components.

Strength of tension tie in UHPC component: $P_n = P_{UHPC} + P_s \leq 1.25 P_s$

$$P_n = 172.8\text{ kips} + (48\text{ ksi})(A_s) \leq 1.25(48\text{ ksi})(A_s)$$

The resistance factor for tension members (specified in UHPC Structural Design Guide Article 1.5.4.2) should be used for the tension tie: $\phi = 0.75$.

Factored strength of tension tie in UHPC component: $\phi P_n = \phi(P_{UHPC} + P_s) \leq \phi 1.25 P_s$

$$\phi P_n = 0.75(172.8\text{ kips} + (48\text{ ksi})(A_s)) \leq (0.75)(1.25)(48\text{ ksi})(A_s)$$

$$\phi P_n = 129.6\text{ kips} + (36\text{ ksi})(A_s) \leq (45\text{ ksi})(A_s)$$

The minimum amount of reinforcement required for UHPC tension elements, as defined by UHPC Structural Design Guide Article 6.6.1, is accounted for by including the upper limit on P_n of $1.25P_s$.

Minimum reinforcement required in UHPC tension elements: $P_s \geq 0.80 P_n$

The same amount of reinforcement will be provided along the bottom of the beam for Tie AB and Tie BC, so the ties will be designed for the maximum demand.

Demand for design of bottom tie: $P_{u,BC}$ = 981.8 kips

The amount of reinforcement required for the design is calculated as follows, initially assuming that the upper limit on P_n of 1.25P_s does not control.

Required amount of reinforcement: $A_{s,req} = (P_{u,BC} - \phi P_{UHPC}) / [\phi(0.80)(f_y)]$

$$A_{s,req} = (981.8 \text{ kips} - 129.6 \text{ kips}) / [(36 \text{ ksi})] = 23.7 \text{ inch}^2$$

The required area of steel is greater than the area of reinforcement initially assumed (A_s = 15.24 inch^2). The required number of No. 10 bars is as follows.

Required No. 10 bars: $n_{req,\#10} = 23.7 \text{ inch}^2 / 1.27 \text{ inch}^2 = 18.6$ bars

The allowable clear distance between bars (specified in UHPC Structural Design Guide Article 10.3 for 0.5-inch fiber length) is 0.75 inches. The maximum number of bars that can fit across the width of the beam is calculated as follows.

Maximum bars across width: $n_{b,max} = [b_{bf} - (2c_c + 2d_{b,trans} - s_c) / (d_{b,long} + s_c)$

$$n_{b,max} = [48 \text{ inch} - (2(0.75\text{inch}) + 2(0.625 \text{ inch}) - 0.75 \text{ inch})] / (1.27 \text{ inch} + 0.75 \text{ inch})$$

$$n_{b,max} = 22.7 \text{ bars}$$

There can be **two layers of 10 No. 10 bars** to give the required area of tension reinforcement in the bottom longitudinal tie. The centroid of the bottom tie remains the same since two layers of No. 10 bars were initially assumed.

Area of steel in Tie AB and Tie BC: $A_{st} = (20 \text{ bars})(1.27 \text{ inch}^2) = 25.4 \text{ inch}^2$

Steel component of strength: $P_s = (48 \text{ ksi})(25.4 \text{ inch}^2) = 1{,}219.2$ kips

The area of the UHPC in the tie region can be calculated deducting the area of the steel as follows.

Tie area for Tie AB and Tie BC: $A_{tie,AB} = A_{tie,BC} = h_a \times b_{bf} - A_{st}$

$$A_{tie,AB} = A_{tie,BC} = (6 \text{ inch})(48 \text{ inch}) - (25.4 \text{ inch}^2) = 262.6 \text{ inch}^2$$

Strength of UHPC in tension tie: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_{tie}$

$$P_{UHPC} = (0.60)(1.0)(1.0 \text{ ksi})(262.6 \text{ inch}^2) = 157.6 \text{ kips}$$

The total tie strength is calculated as follows.

Total tie strength: P_n = 157.6 kips + 1,219.2 kips = 1,377 kips

Factored strength for Tie AB and BC: ϕP_n = 0.75(1,377 kips) = 1,033 kips

This is less than the upper limit on the tie strength, calculated as follows.

Upper limit on tie strength: $\phi P_n \leq \phi 1.25 P_s$ = 0.75(1.25)(1,219.2 kips) = 1,143 kips

The factored strength of Tie AB and BC is greater than the largest demand of the two ties, $P_{u,BC}$ = 981.8 kips.

Tie BD (Vertical Tie)

The total length to include in the vertical tie is equal to the overall shear span length minus a portion taken off for the spread of stresses as follows. This length can be used to calculate the UHPC area in the tie and for detailing the reinforcement in the tie region.

Available length for vertical tie: $\ell_a = a - 2(h_{STM})\tan(25°)$

$$\ell_a = 108 \text{ inch} - 2(66 \text{ inch})\tan(25°) = 46.4 \text{ inch}$$

The total available UHPC area is calculated using this length and the width of the web in the tie region.

Web width in Tie BD: b_w = 3(3 inch) = 9 inch

Initially estimated area of Tie BD: $A_{tie,BD}$ = (9 inch)(46.4 inch) = 418.0 inch^2

This initially estimated tie area does not include the deduction of the steel in the tie region.

The strength of the tie consists of a UHPC component and steel component calculated using the following equations.

Strength of UHPC in tension tie: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_{tie}$

$$P_{UHPC} = (0.60)(1.0)(1.0 \text{ ksi})(418.0 \text{ inch}^2) = 250.8 \text{ kips}$$

Limit on non-prestressed term: $(0.50 E_s \gamma_u \varepsilon_{t,loc}) \leq 0.80 f_y$

$$(0.50)(29{,}000 \text{ ksi})(1.0)(0.005) = 72.5 \text{ ksi} \leq 0.80(60 \text{ ksi}) = 48 \text{ ksi}$$

The upper limit on the stress in the non-prestressed reinforcement, $0.8f_y$, controls for this example with the effective localization strain of 0.005.

Steel Component: $P_s = 0.50 E_s \gamma_u \varepsilon_{t,loc} A_s + A_{ps}[f_{pe} + 0.50 E_s \gamma_u \varepsilon_{t,loc}]$

UHPC Structural Design Guide Eqn. 6.6.1-4

$$P_s = (48 \text{ ksi})(A_s)$$

The amount of reinforcement will be designed using these equations (setting capacity equal to demand), so A_s will be left as a variable at this stage.

The total strength of the tie is the summation of the UHPC and steel components.

Strength of tension tie in UHPC component (with upper limit): $P_n = P_{UHPC} + P_s \leq 1.25 P_s$

$$P_n = 250.8 \text{ kips} + (48 \text{ ksi})(A_s) \leq 1.25(48 \text{ ksi})(A_s)$$

The resistance factor for tension members (specified in UHPC Structural Design Guide Article 1.5.4.2) should be used for the tension tie: $\phi = 0.75$.

Factored strength of tension tie in UHPC component: $\phi P_n = \phi(P_{UHPC} + P_s) \leq \phi 1.25 P_s$

$$\phi P_n = 0.75(250.8 \text{ kips} + (48 \text{ ksi})(A_s)) \leq (0.75)(1.25)(48 \text{ ksi})(A_s)$$

$$\phi P_n = 129.6 \text{ kips} + (36 \text{ ksi})(A_s) \leq (45 \text{ ksi})(A_s)$$

The minimum amount of reinforcement required for UHPC tension elements, as defined by UHPC Structural Design Guide Article 6.6.1, is accounted for by including the upper limit on P_n of $1.25P_s$.

The amount of reinforcement can be calculated based on the demand for Tie BD.

Demand for Tie BD: $P_{u,BD} = 600$ kips

Fifteen sets of three legs of No. 5 bars can be used for the design, with one leg of the No. 5 bar in each of the three webs of the section. The amount of reinforcement required for Tie BD is as follows.

Tie BD Reinforcement: $A_{s,BD} = (15)(3 \text{ legs})(0.31 \text{ inch}^2) = 13.95 \text{ inch}^2$

Steel component of strength: $P_s = (48 \text{ ksi})(13.95 \text{ inch}^2) = 669.6$ kips

The area of the tie region can be calculated deducting the area of the steel as follows.

Area of Tie BD: $A_{tie,BD} = (9 \text{ inch})(46.4 \text{ inch}) - (13.95 \text{ inch}^2) = 404.1 \text{ inch}^2$

Strength of UHPC in tension tie: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_{tie}$

$$P_{UHPC} = (0.60)(1.0)(1.0 \text{ ksi})(404.1 \text{ inch}^2) = 242.4 \text{ kips}$$

This results in the following capacity.

Factored strength of tension tie in UHPC component: $\phi P_n = \phi(P_{UHPC} + P_s) \leq \phi 1.25 P_s$

$$\phi P_n = 0.75(242.4 \text{ kips} + 669.6 \text{ kips}) \leq 0.75(1.25)(669.6 \text{ kips})$$

$$\phi P_n = 684.0 \text{ kips} \leq 627.8 \text{ kips}$$

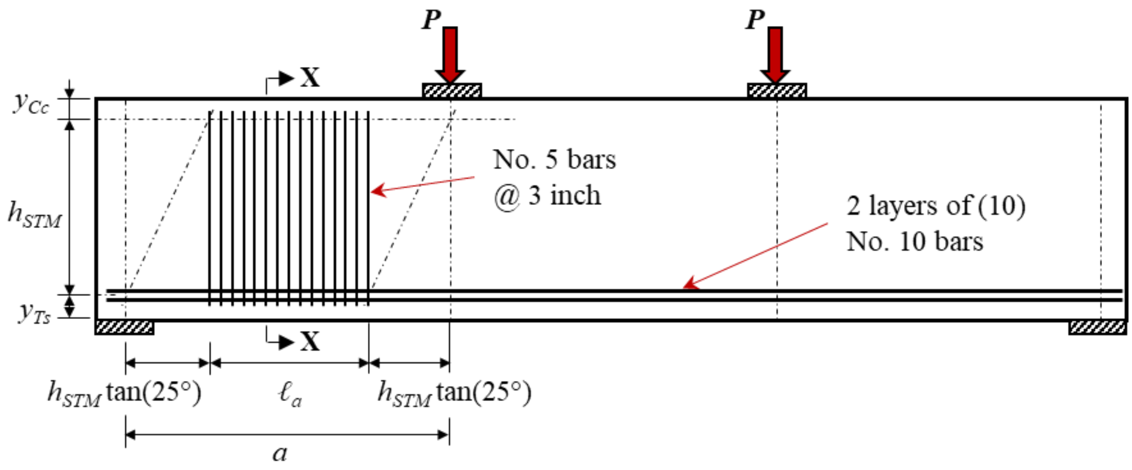
$$\phi P_n = 627.8 \text{ kips} \geq P_{u,BC} = 600 \text{ kips}$$

The 15 sets of No. 5 bars would need to be placed in the length of the tie ($\ell_a = 46.4$ inch), which would lead to the following spacing of the bars in this region.

Spacing of No. 5 bars in Tie BD: $s_{BD} = (46.4 \text{ inch}) / (14 \text{ spaces}) = 3.3$ inch

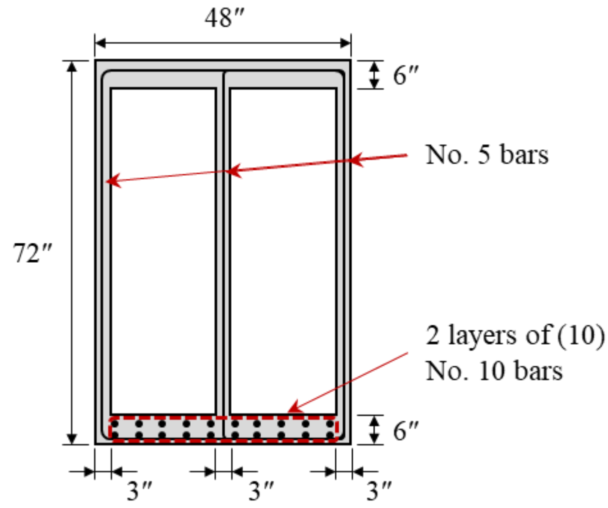
Updated Design of Ties

The final tie configuration is shown in Figure 32 and Figure 33.



Source: FHWA.

Figure 32. Illustration. Elevation view of tie layout.



Source: FHWA.

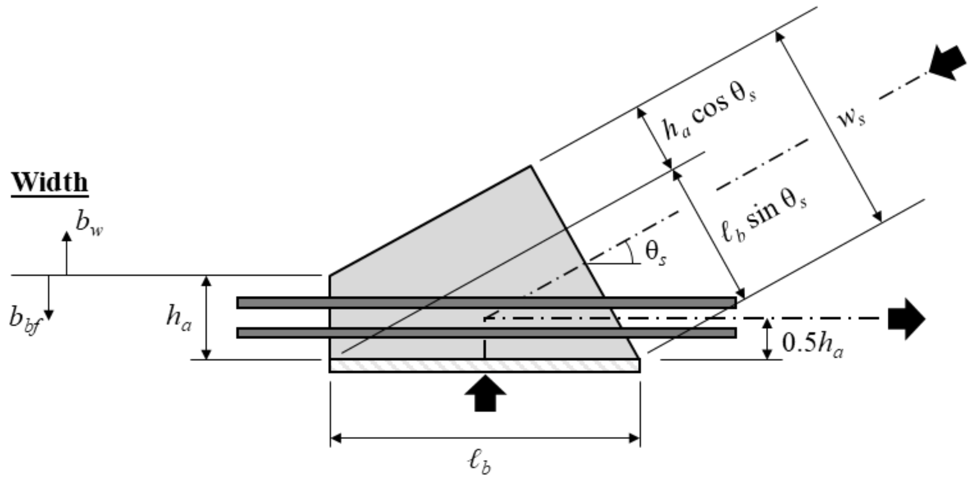
Figure 33. Illustration. Section X-X from Figure 32 with tie reinforcement.

DESIGN STEP 8: PERFORM NODAL STRENGTH CHECKS

There are four nodes with a defined geometry: Node A (CCT), Node C (CCT), Node E (CCC), and Node F (CCC). The other nodes are considered smeared, with undefined geometry, and do not need to be checked. The strength of the bearing face, back face (for CCC nodes), and strut-to-node interface (SNI) needs to be checked during this step. Checking stresses of the back face is not required for CCT nodes with reinforcement without headed reinforcement or an anchor plate.

Node A and Node C (CCT Nodes)

The geometry for the CCT nodes is shown in Figure 34.



Source: FHWA.

Figure 34. Illustration. Geometry of Node A and C (CCT).

The back face height is calculated based on the centroid of the tension tie reinforcement. The centroid of the tension tie reinforcement was calculated in the previous step based on two layers of No. 10 bars for the longitudinal reinforcement, No. 5 transverse bars, 0.75-inch cover, and 0.75-inch spacing between layers of reinforcement.

Back face height for Node A and C: h_a = 6 inch

The length of the bearing is 12 inches for both nodes: ℓ_b = 12 inch. The SNI length, w_s, is calculated based on the strut angle as follows.

SNI length: $w_s = h_a \cos\theta_s + \ell_b \sin\theta_s$

SNI length (Node A): $w_{s,A}$ = (6 inch)cos(50.7°) + (12 inch)sin(50.7°) = 13.1 inch

SNI length (Node C): $w_{s,C}$ = (6 inch)cos(31.4°) + (12 inch)sin(31.4°) = 11.4 inch

The width of the bearing face is equal to the width of the bearing: b_{bf} = 48 inch. The SNI will be assumed to be entirely in the narrower web region: b_w = 9 inch. This will result in the following interface areas.

Bearing face area (Node A and C): A_{cn} = (12 inch)(48 inch) = 576 inch^2

SNI (Node A): A_{cn} = (13.1 inch)(9 inch) = 117.8 inch^2

SNI (Node C): A_{cn} = (11.4 inch)(9 inch) = 102.4 inch^2

The effective concrete strengths to use in the node strength checks are as follows. The confinement modification factor, *m*, is assumed equal to 1.0 for UHPC. The concrete efficiency factors, *v*, of a CCT node in a UHPC component are assumed to be 0.7 for the bearing face and 0.5 for the SNI.

Effective concrete strength (bearing): f_{cu} = (1.0)(0.7)(17.5 ksi) = 12.25 ksi

Effective concrete strength (SNI): f_{cu} = (1.0)(0.5)(17.5 ksi) = 8.75 ksi

The capacity of the bearing face and SNI for Node A and Node C are summarized in Table 5. The demand for each face is also provided to see if the design is sufficient.

Table 5. Summary of bearing face and SNI capacity for Node A and Node C.

Node	Face	A_{cn} (inch^2)	f_{cu} (ksi)	ϕP_n (kips)	P_u (kips	OK?
A	Bearing	576.0	12.25	5,292.0	600.0	OK
A	SNI	117.8	8.75	772.9	775.2	No Good
C	Bearing	576.0	12.25	5,292.0	600.0	OK
C	SNI	102.4	8.75	672.0	1,150.6	No Good

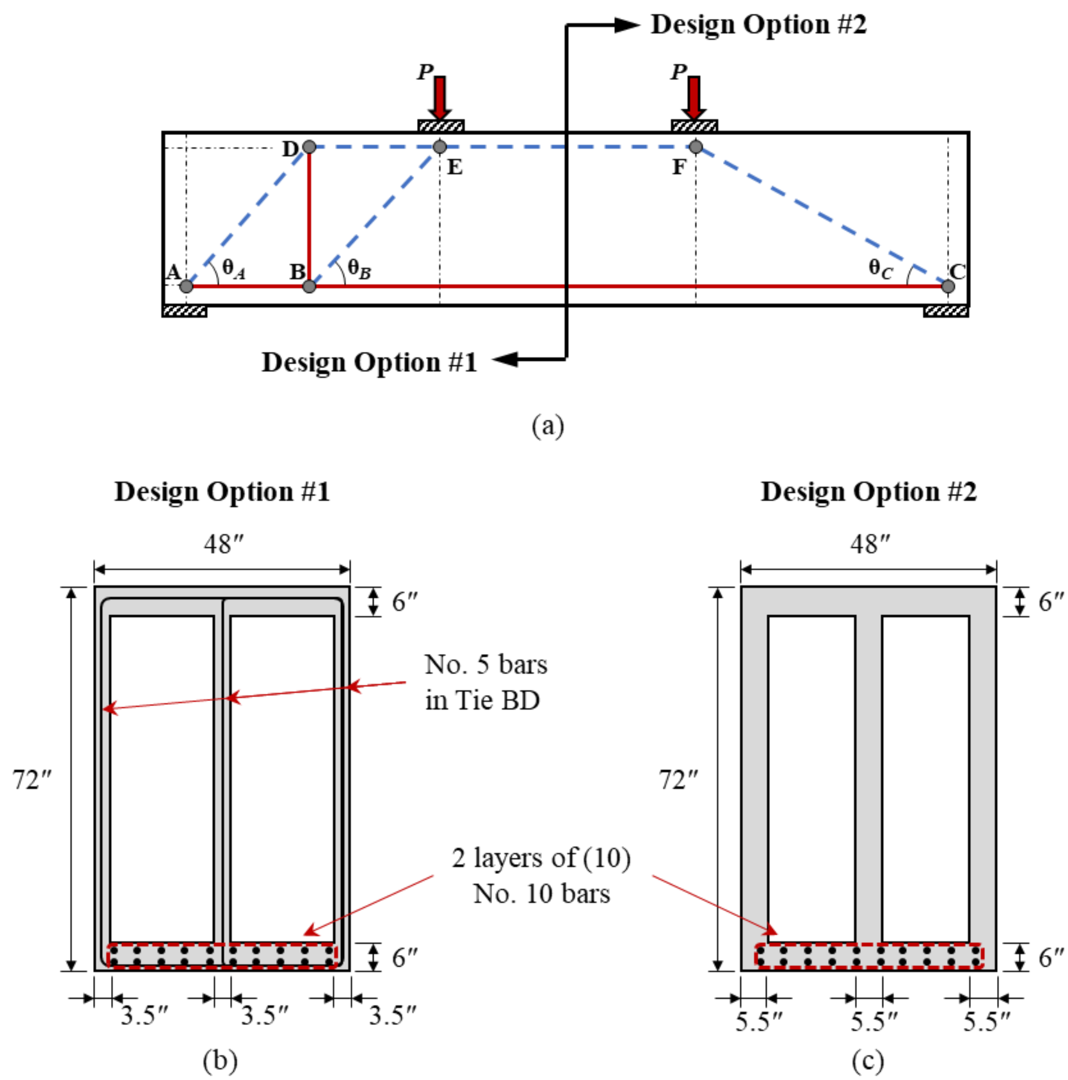
The capacity of the bearing face for both nodes is sufficient.

The capacity of the SNI for both nodes is not sufficient for the current design. There are two ways to increase the strength of the SNI: (1) increase the compressive strength of the UHPC and (2) increase the SNI area by increasing the web width. Both options would likely increase the cost with a higher performance UHPC mixture likely having an increased cost and increasing the web width increasing the volume of UHPC required.

The web width will be increased in this example. For typical designs, the web width should be the same for the entire length of the beam. Two different design options will be considered in this example at this stage, as shown in Figure 35.

- **Design Option #1**: This design option has one intermediate vertical tie between the load and support (Tie BD). The introduction of this intermediate vertical tie creates a two-panel model with a steeper strut angle, which decreases the force in the diagonal strut. This design option would require three 3.5-inch-thick webs in the section at this stage, but would require transverse reinforcement for Tie BD.
- **Design Option #2**: This design option has a direct strut between the load and support. The direct strut leads to a larger force on the SNI than Design Option #1, which requires a larger web width (5.5 inch per web) to support the increased force. However, this design option would not require any transverse reinforcement since there are no vertical ties.

Both design options would have the same amount of reinforcement required for the longitudinal bottom tie.



Source: FHWA.

Figure 35. Illustration. Summary of two different design options based on assumed strut-and-tie model.

The increased SNI areas for each design option are as follows.

SNI Area (Node A, Design Option #1): A_{cn} = (13.1 inch)(10.5 inch) = 137.4 inch^2

SNI Area (Node C, Design Option #2): A_{cn} = (11.4 inch)(16.5 inch) = 187.7 inch^2

The capacity of the SNI for the two options is calculated as follows.

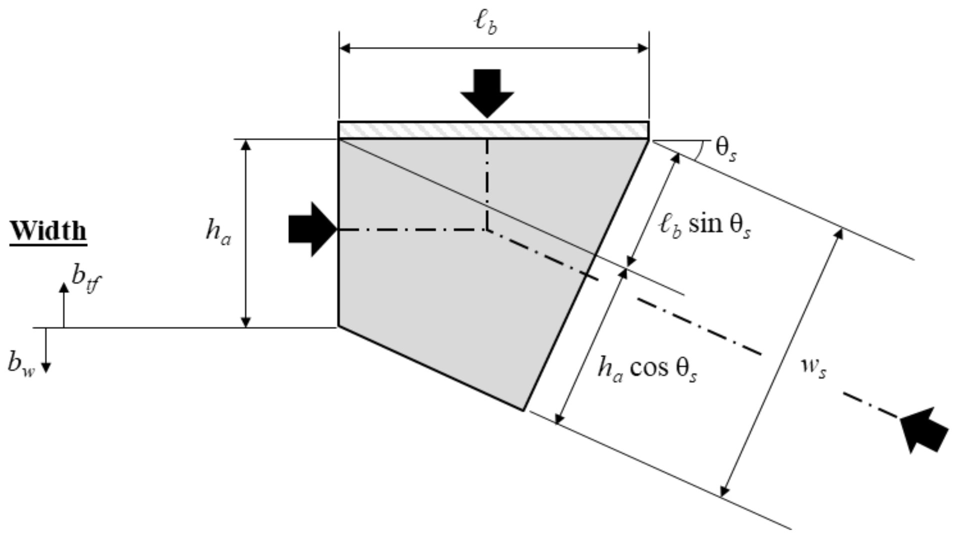
SNI Capacity (Node A, Design Option #1): P_{nA} = (8.75 ksi)(137.4 inch^2) = 1,202 kips
ϕP_{nA} = (0.75)(1,202 kips) = 902 kips $\geq P_u$ = 775 kips → OK

SNI Capacity (Node C, Design Option #2): P_{nC} = (8.75 ksi)(187.7 inch^2) = 1,643 kips
ϕP_{nA} = (0.75)(1,643 kips) = 1,232 kips $\geq P_u$ = 1,151 kips → OK

The capacity of the bearing faces is not affected by the modified web width for the two design options. The capacity of the back face need not be checked for CCT nodes with reinforcement without headed reinforcement or anchor plates.

Node E and Node F (CCC Nodes)

The geometry for the CCC nodes is shown in Figure 36.

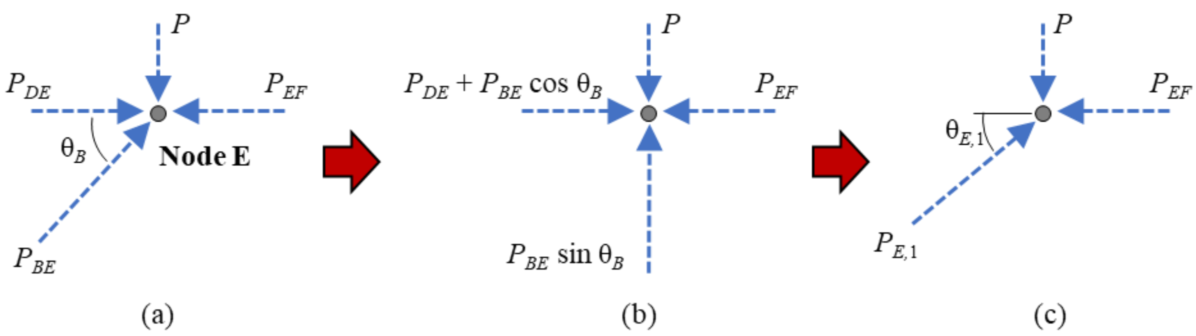


Source: FHWA.

Figure 36. Illustration. Geometry of Node E and F (CCC).

The back face height is calculated based on the centroid of the compression force resultants from the flexure analysis. This is assumed to be the same as previously calculated above: h_a = 6 inch. The length of the bearing is 12 inches for both nodes: ℓ_b = 12 inch.

Node E has four forces intersecting at the node, as shown in Figure 37 (a). These will need to be resolved into three forces for the node strength checks.



Source: FHWA.

Figure 37. Illustration. Resolving two forces into one at Node E.

Two of these intersecting forces should be resolved into one diagonal force. The diagonal force and angle can be calculated based on the vertical forces shown in Figure 37 (b) as follows.

Diagonal resultant force at Node E: $P_{E,1} = \sqrt{[(P_{DE} + P_{BE} \cos \theta_B)^2 + (P_{BE} \sin \theta_B)^2]}$

$P_{E,1} = \sqrt{[(490.9 \text{ kips} + (775.2 \text{ kips})\cos(50.7°))^2 + ((775.2 \text{ kips})\sin(50.7°))^2]}$

$P_{E,1} = 1{,}151$ kips

Angle of resultant force at Node E: $\theta_{E,1} = \tan^{-1}[(P_{BE} \sin \theta_B) / (P_{DE} + P_{BE} \cos \theta_B)]$

$\theta_{E,1} = \tan^{-1}[((775.2 \text{ kips})\sin(50.7°)) / (490.9 \text{ kips} + (775.2 \text{ kips})\cos(50.7°))]$

$\theta_{E,1} = 31.4°$

This diagonal resultant force and angle is used when calculating the SNI length, capacity, and demand. The resultant force and angle are the same as the diagonal strut force and angle for Node F.

The SNI length, w_s, is calculated based on the strut angle of the resultant force as follows.

SNI length: $w_s = h_a \cos \theta_s + \ell_b \sin \theta_s$

SNI length (Node E): $w_{s,E}$ = (6 inch)cos(31.4°) + (12 inch)sin(31.4°) = 11.38 inch

SNI length (Node F): $w_{s,F}$ = (6 inch)cos(31.4°) + (12 inch)sin(31.4°) = 11.38 inch

The width of the bearing face is the width of the bearing: b_{bf} = 48 inch. The SNI will be assumed to be entirely in the narrower web region: $b_{w,E}$ = 10.5 inch and $b_{w,F}$ = 16.5 inch. This will result in the following interface areas.

Bearing face area (Node E and F): A_{cn} = (12 inch)(48 inch) = 576 inch^2

Back face area (Node E and F): A_{cn} = (6 inch)(48 inch) = 288 inch^2

SNI (Node E): A_{cn} = (11.38 inch)(10.5 inch) = 119.5 inch^2

SNI (Node F): A_{cn} = (11.38 inch)(16.5 inch) = 187.7 inch^2

The effective concrete strengths to use in the node strength checks are as follows. The confinement modification factor, m, is assumed equal to 1.0 for UHPC. The concrete efficiency factors, v, of a CCC node in a UHPC component are assumed to be 0.85 for the bearing and back faces and 0.5 for the SNI.

Effective concrete strength (bearing): f_{cu} = (1.0)(0.85)(17.5 ksi) = 14.875 ksi

Effective concrete strength (back face): f_{cu} = (1.0)(0.85)(17.5 ksi) = 14.875 ksi

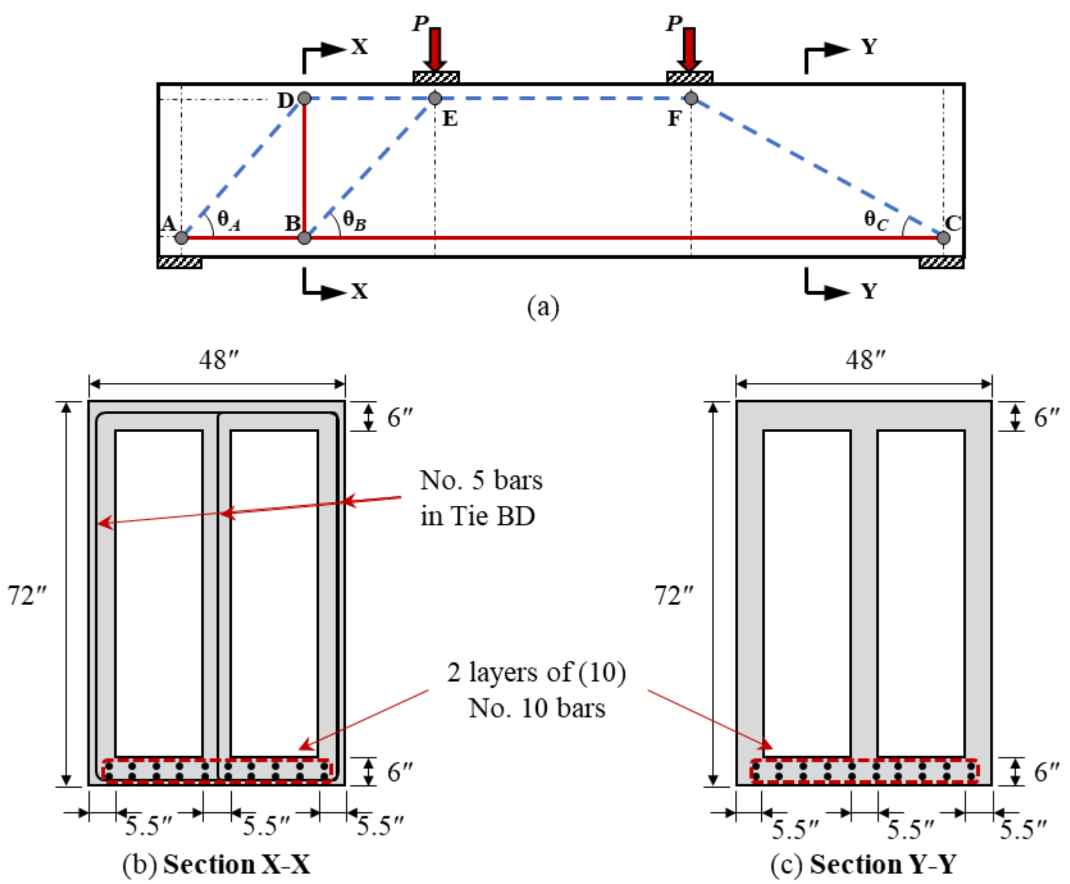
Effective concrete strength (SNI): f_{cu} = (1.0)(0.5)(17.5 ksi) = 8.75 ksi

The capacity of the bearing face, back face, and SNI for Node E and Node F are summarized in Table 6. The demand for each face is also provided to see if the design is sufficient.

Table 6. Summary of bearing face and SNI capacity for Node E and Node F.

Node	Face	A_{cn} (inch^2)	f_{cu} (ksi)	ϕP_n (kips)	P_u (kips)	OK?
E	Bearing	576.0	14.875	6,426.0	600.0	OK
E	Back Face	288.0	14.875	3,213.0	981.9	OK
E	SNI	119.5	8.75	784.0	1,150.6	No Good
F	Bearing	576.0	14.875	6,426.0	600.0	OK
F	Back Face	288.8	14.875	3,213.0	981.9	OK
F	SNI	187.7	8.75	1,231.9	1,150.6	OK

The capacity of the bearing and back faces for both nodes is sufficient. The capacity of the SNI for Node E is not sufficient, so the web width needs to be increased to 5.5 inch per web: b_w = 3(5.5 inch) = 16.5 inch. The updated design based on this increase is shown in Figure 38.



Source: FHWA.

Figure 38. Illustration. Updated design after all nodal stress checks.

The increased SNI area is follows.

SNI Area (Node E): A_{cn} = (11.38 inch)(16.5 inch) = 187.7 inch^2

The capacity of the SNI for Node E is calculated as follows.

SNI Capacity (Node E): P_{nE} = (8.75 ksi)(187.7 inch^2) = 1,643 kips

$\phi P_{n,E,1}$ = (0.75)(1,643 kips) = 1,232 kips ≥ P_u = 1,151 kips → OK

The capacity of the bearing and back faces is not affected by the modified web width for the two design options.

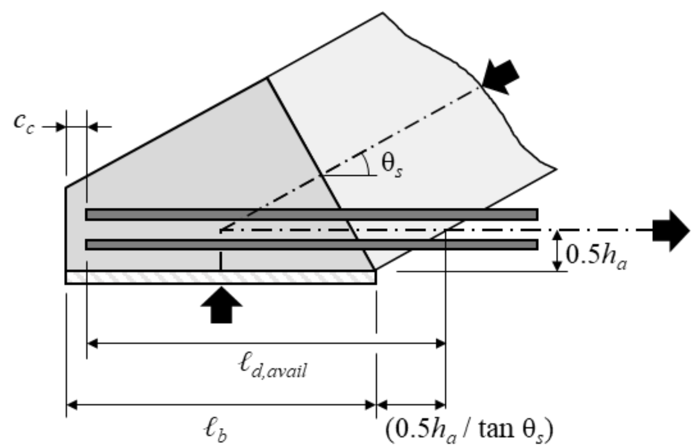
The nodal strength checks are satisfied based on the design shown in Figure 38.

DESIGN STEP 9: PROPORTION CRACK CONTROL REINFORCEMENT

Crack control reinforcement is likely not required for UHPC deep beams, so no additional reinforcement is needed for this design step.

DESIGN STEP 10: PROVIDE NECESSARY ANCHORAGE FOR TIES

The available development length for the reinforcement in Node A and C is from the back of the reinforcement to the point where the reinforcement exits the extended nodal zone, as shown in Figure 39.



Source: FHWA.

Figure 39. Illustration. Details on available development length for Node A and C.

The available development length for Node A and C is calculated as follows.

Available development length: $\ell_{d,avail} = \ell_b - c_c + 0.5h_a / \tan\theta_s$

Available development length (Node A): $\ell_{d,avail,A}$ = 12 inch – 0.75 inch + 0.5(6 inch) / tan(50.7°) = 13.7 inch

Available development length (Node C): $\ell_{d,avail,C}$ = 12 inch – 0.75 inch + 0.5(6 inch) / tan(31.4°) = 16.2 inch

The required development length is calculated using UHPC Structural Design Guide Article 10.8.2. The required development length for No. 9 bars and larger embedded in tension is calculated using AASHTO LRFD BDS Article 5.10.8.2.1 with several amendments. The required development length for No. 10 bars is calculated using these provisions as follows. The compressive strength in this equation is limited to 15 ksi.

Basic development length: $\ell_{db} = 2.4 d_b (f_y / \sqrt{(f'_c)})$ AASHTO LRFD BDS Eqn. 5.10.8.2.1a-2

$$\ell_{db} = 2.4(1.27 \text{ inch})(60 \text{ ksi}) / \sqrt{(15 \text{ ksi})} = 47.2 \text{ inch}$$

There are several factors used to calculate the modified development length as follows.

- Reinforcement location factor: $\lambda_{rl} = 1.0$ (allowed for UHPC)
- Coating factor: $\lambda_{cf} = 1.0$ (for uncoated bars)
- Reinforcement confinement factor: $\lambda_{rc} = 0.917$

  $c_b = 0.75 \text{ inch} + 0.5(1.27 \text{ inch}) = 1.385 \text{ inch}$

  Assume $k_{tr} = 0$ inch (assume $A_{tr} = 0 \text{ inch}^2$)

  $\lambda_{rc} = d_b / (c_b + k_{tr}) = 1.27 \text{ inch} / (1.385 \text{ inch} + 0 \text{ inch}) = 0.917$

- Excess reinforcement factor: $\lambda_{er} = 1.0$ (assume no excess reinforcement provided)
- Concrete density factor: $\lambda = 1.0$ (specified for UHPC)

Modified development length: $\ell_d = \ell_{db} (\lambda_{rl} \times \lambda_{cf} \times \lambda_{rc} \times \lambda_{er}) / \lambda$

AASHTO LRFD BDS Eqn. 5.10.8.2.1-1

$$\ell_d = (47.2 \text{ inch})(1.0)(1.0)(0.917)(1.0) / (1.0) = 43.3 \text{ inch}$$

There is not a sufficient available development length for the straight No. 10 bar embedment: $\ell_{d,avail,A} = 13.7 \text{ inch} < \ell_d = 43.3$ inch. A hooked end can be used to shorten the development length using AASHTO LRFD BDS Article 5.10.8.2.4 (based on UHPC Structural Design Guide Article 10.8.2.4).

Basic development length for hook: $\ell_{hb} = (38 d_b / 60) \times (f_y / \sqrt{(f'_c)})$

AASHTO LRFD BDS Eqn. 5.10.8.2.4a-2

$$\ell_{hb} = (38(1.27 \text{ inch}) / 60) \times (60 \text{ ksi} / \sqrt{(15 \text{ ksi})}) = 12.5 \text{ inch}$$

Modified development length: $\ell_{dh} = \ell_{hb} (\lambda_{rc} \times \lambda_{cw} \times \lambda_{er}) / \lambda$

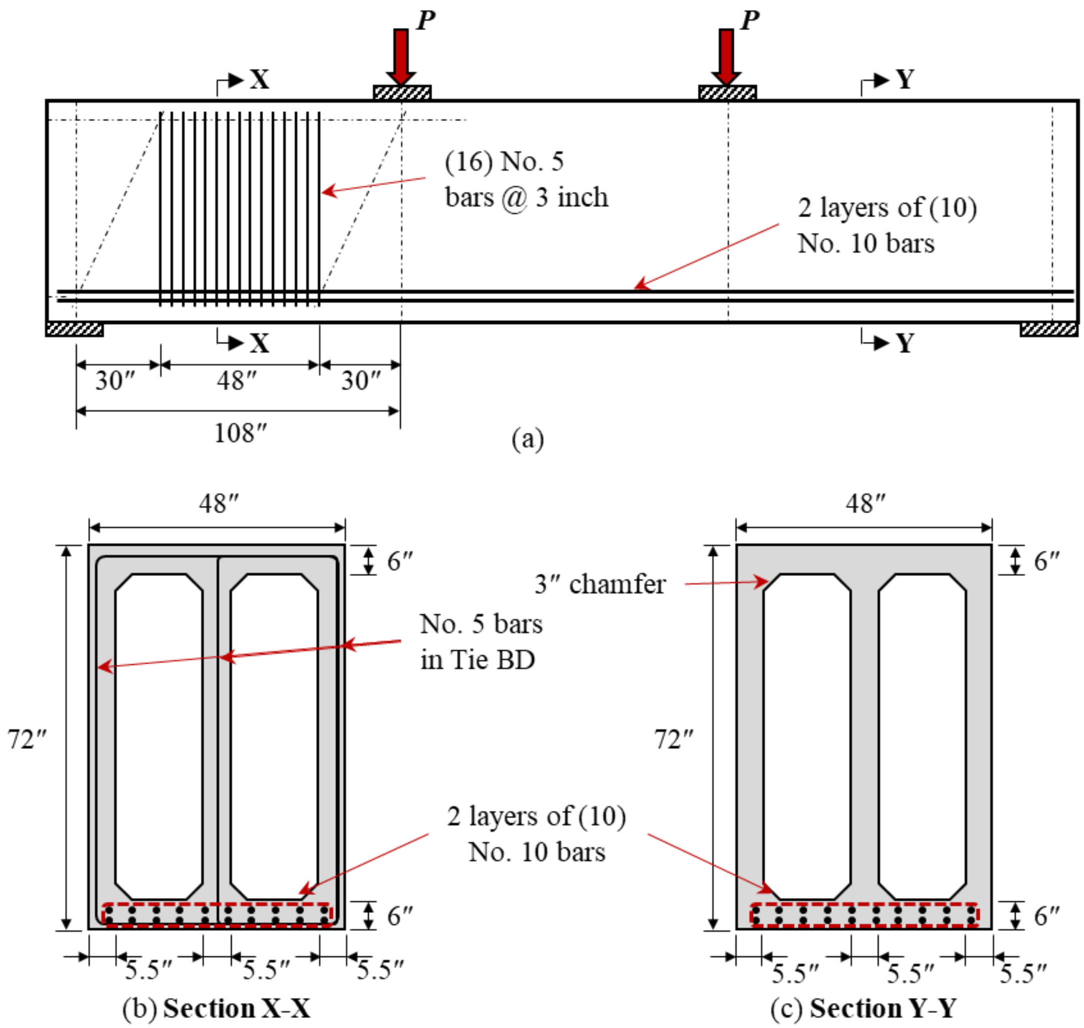
AASHTO LRFD BDS Eqn. 5.10.8.2.4-1

$$\ell_{dh} = (12.5 \text{ inch})(1.0) = 12.5 \text{ inch}$$

The available development lengths for Node A and Node C are greater than the required development length for the hooked reinforcement.

DESIGN STEP 11: DRAW REINFORCEMENT LAYOUT

The reinforcement layout and final cross section details for this example are shown in Figure 40. The amount of transverse reinforcement required is dependent on the strut-and-tie model assumed for each side of the beam. A 3-inch chamfer is included in the void to help avoid stress concentrations and restrained shrinkage cracks that can happen at a sharp corner between the flanges and webs.



Source: FHWA.

Figure 40. Illustration. Final reinforcement layout and cross section design.

SUMMARY FROM EXAMPLE 1

This example showed how the basic framework proposed in this report can be utilized to design a simply-supported UHPC deep beam.

Observations from Model Selection

As with conventional concrete, the strut-and-tie model selected to represent that flow of stresses through a deep beam will affect the final design. Two different model options were used in this example to highlight this difference. The left side of the beam was designed using a two-panel strut-and-tie model with a vertical tie between the load and support and strut angles equal to 50.7 degrees. The right side of the beam was designed using a one-panel model with a strut going directly from the load point to the support; the strut angle for the one panel model was shallower at 31.4 degrees. The two different model assumptions resulted in different designs as described in the following points.

- The one-panel model did not require transverse reinforcement, while the two-panel model required transverse reinforcement in vertical tie region (No. 5 bars spaced at less than 3.3 inches).
- Both the one and two panel models required the same web thickness (16.5 inches).
- The one panel model (with a shallower strut angle) resulted in a larger available development length, due to the larger extended nodal region of the inclined strut.

Based on these observations, the use of a shallower strut angle in the modeling process will result in a more optimized design.

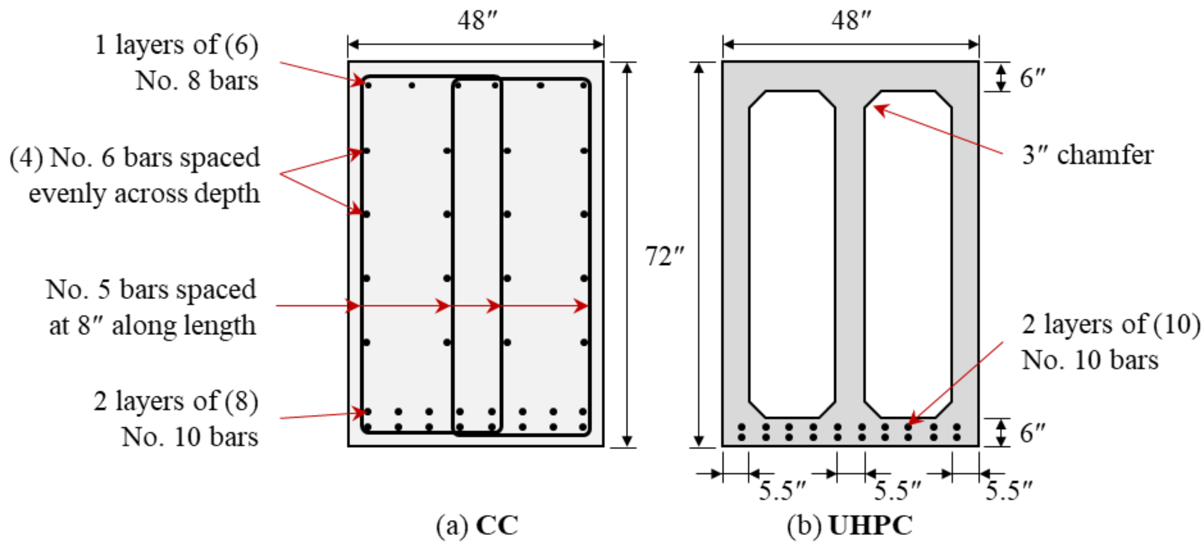
Additional Design Considerations

This strut-and-tie example did not explicitly consider all important aspects of the design of this UHPC deep beam. As summarized in Chapter 3, additional important considerations in this and other designs include the following.

- **Sharp Corners in Voids**: A 3-inch chamfer, which is similar to that often found in box beam voids, was used in this design. This chamfer would help to avoid stress concentrations and restrained shrinkage cracks that can happen at a sharp corner between the flanges and webs. More research is needed to determine the size chamfer required to mitigate these concerns.
- **Direct Load Path from Concentrated Loads into Body of Member**: The load and support bearings extended the full width of the section in this example. Three webs were selected in this design to create a more direct load path from the concentrated loads to the webs. More research is needed related to the required spacing of webs and location of load bearings; like that performed to determine when transverse reinforcement needs to be distributed across the width of the section in AASHTO LRFD BDS Article C5.8.2.5.2.
- **Torsional Resistance**: The load in this example was applied on a bearing that extended the full width of the section and was assumed to be applied symmetrically, so the torsional resistance was not considered.
- **Stability of Member and Section**: The stability was not analyzed in this example problem. However, the design would likely require a solid section at the ends and possibly interior diaphragms.

Comparison with Conventional Concrete Design

A comparison of the basic cross section required for a conventional concrete design (from FHWA-NHI-17-071 Design Example 1) and UHPC design for this example is shown in Figure 41. The UHPC design shown in Figure 41 (b) is based on the single-panel strut-and-tie model, which did not require any transverse reinforcement.



Source: FHWA.

Figure 41. Illustration. Comparison of final design for example using (a) conventional concrete and (b) UHPC.

There were two main advantages of using UHPC in this this design example.

- **Advantage #1 – No Transverse Reinforcement**: The conventional concrete deep beam design required minimum web reinforcement in both the horizontal and vertical directions. The UHPC deep beam design only required longitudinal reinforcement in the bottom tie.
- **Advantage #2 – Lighter Section**: The use of UHPC allowed for two voids to be introduced into the section. This resulted in the UHPC section being approximately 50 percent lighter than the conventional concrete section: $A_{g,CC}$ = 3,456 inch^2 vs. $A_{g,UHPC}$ = 1,602 inch^2 and considering w_c = 0.145 kcf and w_{UHPC} = 0.155 kcf.

The same outside section dimensions as the conventional concrete section were used for the UHPC section in this design example. The section dimensions could have possibly been modified as well to create a shallower section. However, the advantages of a one-panel model would be lost if the strut angle with the bottom tie were to fall below the 25-degree limit.