M 22-24

Highway Engineering Field Formulas



Metric (SI) or US Units

Unless otherwise stated the formulas shown in this manual can be used with any units. The user is cautioned not to mix units within a formula. Convert all variables to one unit system prior to using these formulas.

Significant Digits

Final answers from computations should be rounded off to the number of decimal places justified by the data. The answer can be no more accurate than the least accurate number in the data. Of course, rounding should be done on final calculations only. It should not be done on interim results.

Persons with disabilities may request this information be prepared in alternate forms by calling collect (360) 664-9009. Deaf and hearing impaired people call 1-800-833-6388 (TTY Relay Service).

1998 Engineering Publications Transportation Building Olympia, WA 98504 360-705-7430

CONTENTS

Nomenclature for Circular Curves	2
Circular Curve Equations	4
Simple Circular Curve	5
Degrees of Curvature to Various Radii	6
Nomenclature for Vertical Curves	7
Vertical Curve Equations	8
Nomenclature for Nonsymmetrical Curves	10
Nonsymmetrical Vertical Curve Equations	11
Determining Radii of Sharp Curves	12
Dist. from Fin. Shld. to Subgrade Shld	13
Areas of Plane Figures	14
Surfaces and Volumes of Solids	18
Trigonometric Functions for all Quadrants	23
Trigonometric Functions	24
Right Triangle	25
Oblique Triangle	26
Conversion Factors	28
Metric Conversion Factors	30
Land Surveying Conversion Table	31
Steel Tape Temperature Corrections	31
Temperature Conversion	31
Less Common Conversion Factors	32
Water Constants	32
Cement Constants	32
Multiplication Factor Table	33
Recommended Pronunciations	33
Reinforcing Steel	34

Nomenclature For Circular Curves

- POT Point On Tangent outside the effect of any curve
- POC Point On a circular Curve
- POST Point On a Semi-Tangent (within the limits of a curve)
- PI Point of Intersection of a back tangent and forward tangent
- PC Point of Curvature Point of change from back tangent to circular curve
- PT Point of Tangency Point of change from circular curve to forward tangent
- PCC Point of Compound Curvature -Point common to two curves in the same direction with different radii
- PRC Point of Reverse Curve Point common to two curves in opposite directions and with the same or different radii
- L Total Length of any circular curve measured along its arc
- Lc Length between any two points on a circular curve
- R Radius of a circular curve
- Δ Total intersection (or central) angle between back and forward tangents

Nomenclature For Circular Curves (Cont.)

- DC Deflection angle for full circular curve measured from tangent at PC or PT
- dc Deflection angle required from tangent to a circular curve to any other point on a circular curve
- C Total Chord length, or long chord, for a circular curve
- C Chord length between any two points on a circular curve
- T Distance along semi-Tangent from the point of intersection of the back and forward tangents to the origin of curvature (From the PI to the PC or PT)
- tx Distance along semi-tangent from the PC (or PT) to the perpendicular offset to any point on a circular curve. (Abscissa of any point on a circular curve referred to the beginning of curvature as origin and semi-tangent as axis)
- ty The perpendicular offset, or ordinate, from the semi-tangent to a point on a circular curve
- E External distance (radial distance) from PI to midpoint on a simple circular curve

	_]
Equations	Units
$\mathbf{R} = \frac{\mathbf{180^{\circ}}}{\mathbf{p}} \times \frac{\mathbf{L}}{\mathbf{D}}$	m or ft.
$\mathbf{D} = \frac{180^{\circ}}{\mathbf{p}} \times \frac{\mathbf{L}}{\mathbf{R}}$	degree
$L = \frac{\mathbf{p}}{180} \times \mathbf{R}\mathbf{D}$	m or ft.
$\mathbf{T} = \mathbf{R} \tan \frac{\mathbf{D}}{2}$	m or ft.
$\mathbf{E} = \frac{\mathbf{R}}{\cos\frac{\mathbf{D}}{2}} - \mathbf{R}$	m or ft.
$C = 2R\sin\frac{\mathbf{D}}{2}$, or $= 2R\sin DC$	m or ft.
$MO = R \vec{c} \cdot \cos \frac{D\ddot{o}}{2\dot{b}}$	m or ft.
$DC = \frac{D}{2}$	degree
$dc = \frac{L_c}{L} \frac{a D}{c 2} \ddot{b}$	degree
C' = 2 R sin(dc)	m or ft.
$C = 2R \sin(DC)$	m or ft.
$tx = R \sin(2dc)$	m or ft.
$ty = R[1 - \cos(2dc)]$	m or ft.

Circular Curve Equations

Simple Circular Curve



Constant for $\pi = 3.14159265$

Degree of Curvature for Various Lengths of Radii



Exact for Arc Definition $D = \frac{100\frac{a^2}{c}}{R} = \frac{1800}{pR}$

Where D is Degree of Curvature

Length of Radii for Various Degrees of Curvature





Where R is Radius Length

Nomenclature For Vertical Curves

- G1 & G2 Tangent Grade in percent
- A The absolute of the Algebraic difference in grades in percent
- BVC Beginning of Vertical Curve
- **EVC** End of Vertical Curve
- VPI Vertical Point of Intersection
- L Length of vertical curve
- D Horizontal distance to any point on the curve from BVC or EVC
- E Vertical distance from VPI to curve
- e Vertical distance from any point on the curve to the tangent grade
- **K** Distance required to achieve a 1 percent change in grade
- L₁ Length of a vertical curve which will pass through a given point
- D₀ Distance from the BVC to the lowest or highest point on curve
- X Horizontal distance from P' to VPI
- P and P' Points on tangent grades

Symmetrical Vertical Curve Equations





Notes: All equations use units of length (not stations or increments)

The variable ${\bf A}$ is expressed as an absolute in percent (%)

Example: If $G_1 = +4\%$ and $G_2 = -2\%$ Then A = 6

Symmetrical Vertical Curve Equations (cont.)





Nomenclature For Nonsymmetrical Vertical Curves

- G1 & G2 Tangent Grades in percent
- A The absolute of the Algebraic difference in grades in percent
- **BVC** Beginning of Vertical Curve
- **EVC** End of Vertical Curve
- VPI Vertical Point of Intersection
- I₁ Length of first section of vertical curve
- I2 Length of second section of vertical curve
- L Length of vertical curve
- D₁ Horizontal distance to any point on the curve from BVC towards the VPI
- D₂ Horizontal distance to any point on the curve from EVC towards the VPI
- e1 Vertical distance from any point on the curve to the tangent grade between BVC and VPI
- e₂ Vertical distance from any point on the curve to the tangent grade between EVC and VPI
- E Vertical distance from VPI to curve





 $\mathbf{A} = \left(\mathbf{G}_2\right) - \left(\mathbf{G}_1\right)$

 $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2$

$$\mathbf{E} = \frac{\mathbf{l}_1 \mathbf{l}_2}{200(\mathbf{l}_1 + \mathbf{l}_2)} \mathbf{A}$$

$$\mathbf{e}_{1} = \mathbf{m}_{1}^{\mathbf{\hat{l}}} \frac{\mathbf{D}_{1}}{\mathbf{\hat{l}}_{1}} \mathbf{\hat{\psi}}^{\mathbf{\hat{l}}}$$

$$\mathbf{e}_2 = \mathbf{m}_1^{\mathbf{\hat{l}}} \frac{\mathbf{D}_2}{\mathbf{\hat{l}}_2} \frac{\mathbf{\hat{u}}^2}{\mathbf{\hat{l}}_2} \mathbf{\hat{y}}$$

Determining Radii of Sharp Curves by Field **Measurements**



points on the curve

Example:

Measure the chord length from A to C AC = 18.4 then BC = 9.2Measure the middle ordinate length B to D BD = 3.5

$$\mathbf{R} = \frac{9.2^2}{7.0} + \frac{3.5}{2} = 13.8$$





- **A** = Algebraic difference in % between shld. slope and subgrade slope
- **B** = Depth of surfacing at finished shoulder
- \mathbf{x} = Distance from finished shid. to subgrade shid.

Shoulder	Equivalent	Equivalent
Slope	Rate of Grade	Vertical Angle
1:1.5	66.67%	33°41'24"
1:1.75	57.14%	29°44'42"
1:2	50.00%	26°33'54"
1:2.5	40.00%	21°48'05"
1:3	33.33%	18°26'06"
1:4	25.00%	14°02'10"
1:5	20.00%	11°18'36"
1:6	16.67%	9°27'44"
1:8	12.50%	7°07'30"
1:10	10.00%	5°42'38"
Subgrade	Equivalent	Equivalent
Slope	Rate of Grade	Vertical Angle
.020 / 1	2.00%	1°08'45"
.025 / 1	2.50%	1°25'56"
.030 / 1	3.00%	1°43'06"
.035 / 1	3.50%	2°00'16"
.040 / 1	4.00%	2°17'26"
.050 / 1	5.00%	2°51'45"





Areas of Plane Figures





Areas of Plane Figures

Annulus (Circular Ring)



Irregular Figure



18

Surfaces\Volumes of Solids

Nomenclature

- S Lateral surface area
- ${\bm V} \quad {\rm Volume} \quad$
- A Area of section perpendicular to sides
- B Area of base
- P Perimeter of base
- **P**_A Perimeter of section perpendicular to its sides
- **R** Radius of sphere or circle
- L Slant height or lateral length
- H Perpendicular Height
- **C** Circumference of circle or sphere



Surfaces\Volumes of Solids



Surfaces\Volumes of Solids

Frustum of any Prism or Cylinder



Frustum of Pyramid or Cone Right and Regular, Parallel Ends



Frustum of any Pyramid or Cone, with Parallel Ends









Signs of Trigonometric Functions for All Quadrants



Note:

When using a calculator to compute trigonometric functions from North Azimuths, the correct sign will be displayed

Trigonometric Functions



	$Sinq = \frac{y}{r} = \frac{opposite}{bynotonyco}$	
Sine	r nypotenuse	
	$\cos \mathbf{q} = \frac{\mathbf{x}}{\mathbf{q}} = \frac{\mathrm{adjacent}}{\mathrm{adjacent}}$	
Cosine	r hypotenuse	
	$\tan \mathbf{q} = \frac{\mathbf{y}}{\mathbf{x}} = \frac{\text{opposite}}{\text{adjacent}}$	
Tangent		
	$\cot \mathbf{q} = \frac{\mathbf{x}}{\mathbf{y}} = \frac{\mathrm{adjacent}}{\mathrm{appecite}}$	
Cotangent	y opposite	
	$\sec \mathbf{q} = \frac{\mathbf{r}}{\mathbf{q}} = \frac{\mathbf{hypotenuse}}{\mathbf{r}}$	
Secant	- x adjacent	
	$\csc \mathbf{q} = \frac{\mathbf{r}}{\mathbf{q}} = \frac{\mathbf{hypotenuse}}{\mathbf{q}}$	
Cosecant	y opposite	
	$\sin \mathbf{q} = \frac{1}{2}$ $\tan \mathbf{q} = \frac{1}{2}$	
Reciprocal	$\cos q$	
Relations	$\cos \mathbf{q} = \frac{1}{\sec}$	
	$X = r \times \cos q$	
Rectangular	-	
C	$y = r \times \sin q$	
	$\mathbf{r} = \sqrt{\left(\mathbf{x}^2 + \mathbf{y}^2\right)}$	
Polar	$\mathbf{q} = \arctan \frac{\mathbf{y}}{\mathbf{x}}$	

Right Triangles



A	-B+C=180	0	K=Area
Pythag Theo	gorean Drem	$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$	
A	and B ar	e complementa	ary angles
sin A	= cos B	$\tan A = \cot B$	$\sec A = \csc B$
cos A :	= sin B	cot A = tan B	$\csc A = \sec B$
Given	To Find	Eq	uation
a, c	A, B, b, K	$\sin A = \frac{a}{c}$ $b = \sqrt{c^2 - a^2}$	$\cos B = \frac{a}{c}$ $K = \frac{a}{2}\sqrt{c^2 - a^2}$
a, b	A, B, c, K	$tanA = \frac{a}{b}$ $c = \sqrt{a^2 + b^2}$	$\tan B = \frac{b}{a}$ $K = \frac{ab}{2}$
A, a	B, b, c, K	$B = 90^{\circ} - A$ $c = \frac{a}{\sin A}$	$b = a > \cot A$ $k = \frac{a^2 \times \cot A}{2}$
A, b	B, a, c, K	$B = 90^{\circ} - A$ $c = \frac{b}{\cos A}$	$a = b \times \tan A$ $K = \frac{b^2 \times \tan A}{2}$
A, c	B, a, b, K	$B = 90^{\circ} - A$ $b = c > \cos A$	$a = c > \sin A$ $K = \frac{c^2 \times \sin 2A}{4}$

Oblique Triangles



Law o	f Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Law of	Cosines	$a2 = b2 + c2 - 2bc \times cos A$ $b2 = a2 + c2 - 2ac \times cos B$
		$c^2 = a^2 + b^2 - 2ab \times cos C$
Sum of	Angles	$\mathbf{A} + \mathbf{B} + \mathbf{C} = 180^{0}$
K =	Area	$s = \frac{a+b+c}{2}$
Given	To Find	Equation
		$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
a, b, c	Α	$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

		de mangles
Given	To Find	Equation
a, b, c	В	$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$ $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$ $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$
a, b, c	С	$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$ $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$ $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$
a, b, c	K	$K = \sqrt{s(s - a)(s - b)(s - c)}$
a, A, B	b, c	$b = \frac{a \times \sin B}{\sin A}$ $c = \frac{a \times \sin(A + B)}{\sin A}$
a, A, B	К	$K = \frac{ab \times \sin C}{2} = \frac{a^2 \times \sin B \times \sin C}{2 \times \sin A}$
a, b, A	В	$\sin B = \frac{b \times \sin A}{a}$
a, b, A	c	$c = \frac{a \times \sin C}{\sin A} = \frac{b \times \sin C}{\sin B}$ $c = \sqrt{a^2 + b^2 - 2ab \times \cos C}$
a, b, A	К	$K = \frac{ab > sin C}{2}$
a, b, C	Α	$\tan A = \frac{a \times \sin C}{b - a \times \cos C}$
a, b, C	с	$c = \frac{a \times \sin(A + B)}{\sin A}$ $c = \sqrt{\left(a^2 + b^2 - 2ab \times \cos C\right)}$
a, b, C	K	$\mathbf{K} = \frac{\mathbf{ab} \cdot \sin \mathbf{C}}{2}$

Oblique Triangles

Class	multiply:	by:	to get:
Length	in	0.0833	ft
	in	0.028	yd
	ft	12	in
	ft	0.33	yd
	ft	0.06	rods
	yd	36	in
	yd	3	ft
	yd	0.18	rods
	rods	198	in
	rods	16.5	ft
	rods	5.5	yd
	mi	5280	ft
	mi	1760	yd
	mi	320	rods
Area	in ²	0.007	ft ²
	ft ²	144	in ²
	ft ²	0.11	vd ²
	yd ²	1296	in ²
	yd ²	9	ft
	vď	0.03	rods ²
	rods [∠]	272.25	ft
	rods ²	30.25	vd ²
	acres	43560	ft ²
	acres	4840	yd [∠]
	acres	160	rods ²

Conversion Factors

Class	multiply:	by:	to get:
Volume	ft ³	1728	in ³
	ft ³	0.04	yd ³
	ft ³	7.48	gallons
	yd ³	27	ft ³
	yd ³	202	gallons
	quarts	2	pints
	quarts	0.25	gallons
	gallons	8	pints
	gallons	4	quarts
	gallons	0.13	ft ³
Force	ounces	0.06	pounds
	pounds	16	ounces
	tons (short)	2000	pounds
	tons	2205	pounds
	(metric)		
Velocity	miles/hr	88	ft/min
	miles/hr	1.47	ft/sec

Conversion Factors

.

Class	multiply:	by:	to get:
Length	in	25.40	mm
	in	2.540	cm
	in	0.0254	m
	ft	0.3048	m
	yd	0.9144	m
	mi	1.6093	km
Area	ft ²	0.0929	m ²
	yd ²	0.8361	m ²
	mi ²	2.590	km ²
Volume	in ³	16.387	cm ³
	ft ³	0.0283	m ³
	yd ³	0.7646	m ³
	gal	3.785	L
	gal	0.0038	m ³
	fl oz	29.574	mL
	acre ft	1233.48	m ³
Mass	oz	28.35	g
	lb	0.4536	kg
	kip	0.4536	metric ton
	(1000 lb)		(1000 kg)
	short ton	907.2	kg
	2000 lb		
	short ton	0.9072	metric ton

Metric Conversion Factors

Land Surveying Conversion Factors

Class	multiply:	by:	to get:
Area	acre	4046.8726	m ²
	acre	0.40469	ha
			10000 m ²
Length	ft	12/39.37*	m

* Exact, by definition of the U.S. Survey foot

Steel Tape Temperature Corrections

$$\begin{split} C = & 11.66 \cdot 10^{-6} \big(T_C - 20 \big) L_m \\ & \text{or} \\ C = & 6.45 \cdot 10^{-6} \big(T_F - 68 \big) L_f \\ & \text{Where:} \end{split}$$

- **C** = Correction
- T_c = Temperature in degrees Celsius
- L_M = Length in meters
- T_F = Temperature in degrees Fahrenheit
- L_f = Length in feet

Temperature Conversion

Fahrenheit to Celsius

Celsius to Fahrenheit

$$\frac{3}{9}(^{\circ}F - 32)$$
$$\left(\frac{9}{5} ^{\circ}C\right) + 32$$

Less Common Conversion Factors

Class	multiply:	by:	to get:
Density	lb/ft ³	16.0185	kg/m ³
	lb/yd ³	0.5933	kg/m ³
Pressure	psi	6894.8	Ра
	ksi	6.8948	MPa
	lb/ft ²	47.88	Ра
Velocity	ft/s	0.3048	m/s
	mph	0.4470	m/s
	mph	1.6093	km/h

Water Constants

Freezing point of water = 0° C (32° F) Boiling point of water under pressure of one atmosphere = 100° C (212° F)

The mass of one cu. meter of water is 1000 kg The mass of one liter of water is 1 kg (2.20 lbs)

1 cu. ft. of water $@60^{\circ}$ F = 62.37 lbs (28.29 kg) 1 gal of water $@60^{\circ}$ F = 8.3377 lbs (3.78 kg)

Cement Constants

1 sack of cement (appx.) = 1 ft^3 = 0.028 m³

- 1 sack of cement = 94 lbs. = 42.64 kg
- 1 gallon water = 8.3453 lbs. @39.2° F
- 1 gallon water = $3.7854 \text{ kg} @4^{\circ} \text{ C}$

Multiple	Prefix	Symbol
$1\ 000\ 000\ 000 = 10^9$	giga	G
$1\ 000\ 000 = 10^6$	mega	М
$1\ 000 = 10^3$	kilo	k
$100 = 10^{2}$	*hecto	h
$10 = 10^{1}$	*deka	da
$0.1 = 10^{-1}$	*deci	d
$0.01 = 10^{-2}$	*centi	С
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001\ =\ 10^{-9}$	nano	n

Multiplication Factor Table

* Avoid when possible

Recommended Pronunciations

Prefix	Pronunciation		
giga	jig'a (i as in <i>jig,</i> a as in <i>a</i> -bout		
mega	as in <i>mega</i> -phone		
kilo	kill' oh		
hecto	heck' toe		
deka	deck' a (a as in <i>a</i> -bout		
centi	as in <i>centi</i> -pede		
milli	as in <i>mili</i> -tary		
micro	as in <i>micro</i> -phone		
nano	nan' oh		

Bar	Nominal	Nominal	Unit Weight
Size	Diameter	Area	
#3	9.5mm	71mm ²	0.560kg\m
	[0.375 in]	[0.110 in ²]	[0.376 lb\ft]
#4	12.7mm	127mm ²	0.994kg\m
	[0.500 in]	[0.197 in ²]	[0.668 lb\ft]
#5	15.9mm	199mm ²	1.552kg∖m
	[0.625 in]	[0.309 in ²]	[1.043 lb∖ft]
#6	19.1mm	287mm ²	2.235kg\m
	[0.750 in]	[0.445 in ²]	[1.502 lb\ft]
#7	22.2mm	387mm ²	3.045kg∖m
	[0.875 in]	[0.600 in ²]	[2.044 lb∖ft]
#8	25.4mm	507mm ²	3.973kg∖m
	[1.000 in]	[0.786 in ²]	[2.670 lb∖ft]
#9	28.7mm	647mm ²	5.060kg\m
	[1.128 in]	[1.003 in ²]	[3.400 lb\ft]
#10	32.3mm	819mm ²	6.404kg\m
	[1.270 in]	[1.270 in ²]	[4.303 lb\ft]
#11	35.8mm	1007mm ²	7.907kg\m
	[1.410 in]	[1.561 in ²]	[5.313 lb\ft]
#14	43.0mm	1452mm ²	11.384kg\m
	[1.693 in]	[2.251 in ²]	[7.650 lb\ft]
#18	57.3mm	2579mm ²	20.239kg\m
	[2.257 in]	[3.998 in ²]	[13.600 lb\ft]

Reinforcing Steel

Notes

Notes